Problem 1 (10 points)

(a) If a pendulum clock is accurate at sea level, will it gain or lose time when taken to high altitude? Explain.

ANSWER: The period of the clock is

$$T = 2\pi \sqrt{\frac{L}{g}}$$

At high altitude, $g$ is less than at sea level, so the period is longer, so the clock will lose time.

(b) If you dangle two pieces of paper vertically, a few inches apart, and blow between them, how will they move? Explain using Bernoulli’s principle.

ANSWER: Before you blow, the pressure is $P_{atm}$ on both sides of each piece of paper. When you blow, the pressure in between becomes $P$ and by Bernoulli’s principle

$$P + \frac{1}{2} \rho v^2 = P_{atm}$$

So $P$ is now less and there is a pressure difference and therefore a net force

$$F = (P_{atm} - P)A = \frac{1}{2} \rho v^2 A$$

forcing the two pieces of paper to move toward each other.

(c) Explain why it is not possible to sit upright in a chair and rise to your feet without first leaning forward. Draw a force diagram to support your explanation.

ANSWER: When you are sitting, the torque of your weight about your feet is counter-balanced by the torque from the chair. To get up, you need to create an opposite torque about your feet, which is what happens when you lean forward.

(d) When a sound wave passes from air into water, how will its frequency and wavelength change? Explain.

ANSWER: The frequency depends on the source of the wave, so it will not change. The speed of the wave is

$$v = \lambda f$$

In the water, sound travels faster than in the air, so the wavelength $\lambda = v/f$ will be longer.
Problem 2 (10 points)
When you attach a 5 kg ball to a vertical spring, the spring stretches by 12 cm.

(a) What is the spring constant?
\[ F = -kx, \quad F = -W = -mg = 5 \times 9.8 \, N = 49 \, N \]
so
\[ k = \frac{W}{x} = \frac{49 \, N}{12 \times 10^{-2} \, m} = 408.3 \, N/m \]

(b) If you pull the ball down 4 cm more and then release it so that it vibrates up and down, what will be the amplitude, period and frequency of vibration?
\[ A = 4 \, cm \]
\[ T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{5}{408.3}} \, s = 0.7 \, s \]
\[ f = \frac{1}{T} = 1.4 \, Hz \]

(c) What will be the maximum speed of the ball?
\[ v_0 = A\omega = 4 \times 10^{-2} \times 2\pi \times 1.4 \, m/s = 0.36 \, m/s \]

Problem 3 (10 points)
You apply a 3 N force to the plunger of a hypodermic needle. The diameter of the plunger is 1.5 cm and that of the needle is 0.18 mm.

(a) What is the pressure in the fluid?
\[ p = \frac{F}{A_{plunger}} = \frac{3 \, N}{\pi(1.5 \times 10^{-2}m/2)^2} = 1.7 \times 10^{4} N/m^2 \]

(b) With what force does the fluid leave the needle?
\[ F_{needle} = pA_{needle} = 1.7 \times 10^{4} N/m^2 \times \pi(0.18 \times 10^{-3}m/2)^2 = 4.3 \times 10^{-4} N \]

(c) What force on the plunger would be needed to push fluid into a vein where the pressure is 20 mm-Hg? [1 mm-Hg = 133 N/m²]
\[ F = pA_{plunger} = 20 \times 133 \, N/m^2 \times \pi(1.5 \times 10^{-2}m/2)^2 = 0.47 \, N \]
(d) If you are moving the plunger with speed 2 mm/s, with what speed does the fluid leave the needle?

ANSWER:

\[ A_{\text{plunger}}v_{\text{plunger}} = A_{\text{needle}}v_{\text{needle}} \]

so

\[ v_{\text{needle}} = \frac{A_{\text{plunger}}}{A_{\text{needle}}} v_{\text{plunger}} = \frac{\pi (1.5 \times 10^{-2} m/2)^2}{\pi (0.18 \times 10^{-3} m/2)^2} \times 2 \times 10^{-3} m/s = 13.9 m/s \]

Problem 4 (10 points)

A 17 cm long animal tendon was found to stretch 3.6 mm by a force of 15 N. The tendon was approximately round with an average diameter of 8.5 mm.

(a) What is the stress, strain and elastic modulus of this tendon?

ANSWER:

\[ \text{stress} = \frac{F}{A} = \frac{15 N}{\pi (8.5 \times 10^{-3} m/2)^2} = 2.67 \times 10^5 N/m^2 \]

\[ \text{strain} = \frac{\Delta L}{L} = \frac{3.6 \times 10^{-3} m}{17 \times 10^{-2} m} = 0.02 \]

\[ E = \frac{\text{stress}}{\text{strain}} = \frac{2.67 \times 10^5 N/m^2}{0.02} = 1.26 \times 10^7 N/m^2 \]

(b) If a 40 N force is applied to this tendon instead, how much will it stretch?

ANSWER: Solving for strain,

\[ \text{strain} = \frac{\text{stress}}{E} = \frac{F/A}{E} = \frac{40}{\pi (8.5 \times 10^{-3} / 2)^2 \times 1.32 \times 10^7} = 0.056 \]

By the definition of strain,

\[ \Delta L = \text{strain} \times L = 0.056 \times 17 \text{ cm} = 0.96 \text{ cm} = 9.6 \text{ mm} \]