Problem 1 (10 points)

(a) If you dangle two pieces of paper vertically, a few inches apart, and blow between them, how will they move? Explain using Bernoulli’s principle.

ANSWER: Before you blow, the pressure is $P_{atm}$ on both sides of each piece of paper. When you blow, the pressure in between becomes $P$ and by Bernoulli’s principle

$$P + \frac{1}{2} \rho v^2 = P_{atm}$$

So $P$ is now less and there is a pressure difference and therefore a net force

$$F = (P_{atm} - P)A = \frac{1}{2} \rho v^2 A$$

forcing the two pieces of paper to move toward each other.

(b) When a sound wave passes from air into water, how will its frequency and wavelength change? Explain.

ANSWER: The frequency depends on the source of the wave, so it will not change. The speed of the wave is

$$v = \lambda f$$

In the water, sound travels faster than in the air, so the wavelength $\lambda = v/f$ will be longer.

(c) If a pendulum clock is accurate at sea level, will it gain or lose time when taken to high altitude? Explain.

ANSWER: The period of the clock is

$$T = 2\pi \sqrt{\frac{L}{g}}$$

At high altitude, $g$ is less than at sea level, so the period is longer, so the clock will lose time.

(d) Explain why it is not possible to sit upright in a chair and rise to your feet without first leaning forward. Draw a force diagram to support your explanation.

ANSWER: When you are sitting, the torque of your weight about your feet is counterbalanced by the torque from the chair. To get up, you need to create an opposite torque about your feet, which is what happens when you lean forward.
Problem 2 (10 points)
A 17 cm long animal tendon was found to stretch 3.6 mm by a force of 15 N. The tendon was approximately round with an average diameter of 8.5 mm.

(a) What is the stress, strain and elastic modulus of this tendon?

\[
\text{stress} = \frac{F}{A} = \frac{15 \text{ N}}{\pi (8.5 \times 10^{-3} \text{m}/2)^2} = 2.67 \times 10^5 \text{N/m}^2
\]

\[
\text{strain} = \frac{\Delta L}{L} = \frac{3.6 \times 10^{-3} \text{m}}{17 \times 10^{-2} \text{m}} = 0.02
\]

\[
E = \frac{\text{stress}}{\text{strain}} = \frac{2.67 \times 10^5 \text{N/m}^2}{0.02} = 1.26 \times 10^7 \text{N/m}^2
\]

(b) If a 40 N force is applied to this tendon instead, how much will it stretch?

ANSWER: Solving for strain,

\[
\text{strain} = \frac{\text{stress}}{E} = \frac{F/A}{E} = \frac{40}{\pi (8.5 \times 10^{-3} / 2)^2 \times 1.32 \times 10^7} = 0.056
\]

By the definition of strain,

\[
\Delta L = \text{strain} \times L = 0.056 \times 17 \text{ cm} = 0.96 \text{ cm} = 9.6 \text{ mm}
\]

Problem 3 (10 points)
When you attach a 5 kg ball to a vertical spring, the spring stretches by 12 cm.

(a) What is the spring constant?

ANSWER:

\[
F = -kx, \quad F = -W = -mg = 5 \times 9.8 \text{ N} = 49 \text{ N}
\]

so

\[
k = \frac{W}{x} = \frac{49 \text{ N}}{12 \times 10^{-2} \text{m}} = 408.3 \text{ N/m}
\]

(b) If you pull the ball down 4 cm more and then release it so that it vibrates up and down, what will be the amplitude, period and frequency of vibration?

ANSWER:

\[
A = 4 \text{ cm}
\]

\[
T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{5}{408.3}} \quad s = 0.7 \text{ s}
\]

\[
f = \frac{1}{T} = 1.4 \text{ Hz}
\]
(c) What will be the maximum speed of the ball?

\[ v_0 = A\omega = 4 \times 10^{-2} \times 2\pi \times 1.4 \text{ m/s} = 0.36 \text{ m/s} \]

Problem 4 (10 points)

You apply a 3 N force to the plunger of a hypodermic needle. The diameter of the plunger is 1.5 cm and that of the needle is 0.18 mm.

(a) What is the pressure in the fluid?

**ANSWER:**

\[ p = \frac{F}{A_{\text{plunger}}} = \frac{3 \text{ N}}{\pi (1.5 \times 10^{-2} \text{m}/2)^2} = 1.7 \times 10^4 \text{ N/m}^2 \]

(b) With what force does the fluid leave the needle?

**ANSWER:**

\[ F_{\text{needle}} = pA_{\text{needle}} = 1.7 \times 10^4 \text{ N/m}^2 \times \pi (0.18 \times 10^{-3} \text{m}/2)^2 = 4.3 \times 10^{-4} \text{ N} \]

(c) What force on the plunger would be needed to push fluid into a vein where the pressure is 20 mm-Hg? [1 mm-Hg = 133 N/m²]

**ANSWER:**

\[ F = pA_{\text{plunger}} = 20 \times 133 \text{ N/m}^2 \times \pi (1.5 \times 10^{-2} \text{m}/2)^2 = 0.47 \text{ N} \]

(d) If you are moving the plunger with speed 2 mm/s, with what speed does the fluid leave the needle?

**ANSWER:**

\[ A_{\text{plunger}}v_{\text{plunger}} = A_{\text{needle}}v_{\text{needle}} \]

so

\[ v_{\text{needle}} = \frac{A_{\text{plunger}}}{A_{\text{needle}}} v_{\text{plunger}} = \frac{\pi (1.5 \times 10^{-2} \text{m}/2)^2}{\pi (0.18 \times 10^{-3} \text{m}/2)^2} \times 2 \times 10^{-3} \text{ m/s} = 13.9 \text{ m/s} \]