Problem 1 (10 points)

(a) Two identical arrows, one with four times the speed of the other, are fired into a bale of hay. Assuming the hay exerts a constant frictional force on the arrows, the faster arrow will penetrate how much farther than the slower arrow? Explain.

ANSWER: The kinetic energy is equal to the work done by friction, 
\[ K.E. = Fd \]

Since \( K.E. = \frac{1}{2}mv^2 \), if \( v_2 = 4v_1 \), then \( K.E.2 = 16K.E.1 \), and so 
\[ d_2 = 16d_1 \]

(b) If the net force on a system is zero, is the net torque also zero? Draw a picture to support your answer.

ANSWER: Consider two opposite forces, \( F \) and \( -F \) applied at distances \( d \) on different sides of point of rotation. Then total force 
\[ F_{TOTAL} = F - F = 0 \]

but total torque is 
\[ T_{TOTAL} = 2Fd \]

(c) Two balls have the same kinetic energy but one is twice as heavy as the other. What is the ratio of their respective momenta? Which ball has the greater momentum?

ANSWER: Given \( m_2 = 2m_1 \), \( K.E.2 = K.E.1 \), we have \( 2v_2^2 = v_1^2 \). Ratio of momenta:
\[ \frac{p_2}{p_1} = \frac{m_2v_2}{m_1v_1} = 2 \times \frac{1}{\sqrt{2}} = \sqrt{2} = 1.41 \]

so \( p_2 > p_1 \) (heavier ball has greater momentum).

(d) A car is moving at constant speed along a mountain road. Is the force it exerts on the road at the top of a hill greater or less than its weight? Draw a force diagram to justify your answer.

ANSWER: The force that the car exerts on the road is opposite to the normal force \( N \). The total force is \( F = W - N \). At the top of the hill, this is the centripetal force, so
\[ W - N = m \frac{v^2}{r} \]
and so \( N = W - m \frac{v^2}{r} \)

Therefore, \( N < W \).
Problem 2 (10 points)

(a) What is the distance from the Earth’s center to a point outside the Earth where the gravitational acceleration due to the Earth is half its value at the Earth’s surface (0.5g)? The Earth’s radius is 6,400 km.

ANSWER: We know
\[ F = G \frac{mM}{r^2} \]

Acceleration is
\[ a = \frac{F}{m} = \frac{GM}{r^2} \]

At the Earth’s surface, \( r = R_E = 6,400 \) km, we have
\[ a = g = \frac{GM}{R_E^2} \]

Let \( a = 0.5g \) at distance \( r \). Then
\[ r^2 = \frac{GM}{a} = \frac{GM}{0.5g} = \frac{GM}{0.5 \times \frac{GM}{R_E^2}} = 2R_E^2 \]

and so
\[ r = \sqrt{2}R_E = 1.41 \times 6,400 \text{ km} = 9051 \text{ km} \]

(b) If a satellite is orbiting the Earth at that distance, what is its speed \( v \) and period \( T \) of revolution?

ANSWER: If the satellite is orbiting, its acceleration \( a = 0.5g \) is centripetal acceleration, so
\[ a = 0.5g = \frac{v^2}{r} \]

and
\[ v = \sqrt{0.5gr} = \sqrt{0.5 \times 9.8 \times 9.051 \times 10^6} \text{ m/s} = 6.7 \times 10^3 \text{ m/s} \]

For the period, use \( v = 2\pi r/T \),
\[ T = \frac{2\pi r}{v} = \frac{2\pi \times 9.051 \times 10^6}{6.7 \times 10^3} \text{ s} = 8,488 \text{ s} = 2.36 \text{ days} \]

Problem 3 (10 points)
A 50,000-kg train travels on a level frictionless track with a constant speed of 25 m/s. Suddenly, a 10,000-kg additional load is dumped into the train.
(a) **What then will be the train’s speed?**

**ANSWER:** Conservation of momentum:

\[ Mv = Mv' + mv' \]

where \( M \) is the mass of the train, \( m \) the mass of the package and \( v \) (\( v' \)) the speed of the train before (after) the collision.

Solving for \( v' \),

\[ v' = \frac{Mv}{M + m} = \frac{50,000 \times 25}{50,000 + 10,000} \text{ m/s} = 20.83 \text{ m/s} \]

(b) **What will be the kinetic energy of the system (including the additional load)?**

**ANSWER:**

\[ K.E. = \frac{1}{2}(M + m)v'^2 = \frac{1}{2} \times (50,000 + 10,000) \times 20.83^2 \text{ J} = 1.30 \times 10^7 \text{ J} \]

(c) **How much heat was generated?**

**ANSWER:** The kinetic energy before the collision is

\[ K.E._{\text{BEFORE}} = \frac{1}{2}Mv^2 = \frac{1}{2} \times 50,000 \times 25^2 \text{ J} = 1.56 \times 10^7 \text{ J} \]

Energy lost (heat) is

\[ Q = K.E._{\text{BEFORE}} - K.E. = 1.56 \times 10^7 \text{ J} - 1.30 \times 10^7 \text{ J} = 2.6 \times 10^6 \text{ J} \]

**Problem 4 (10 points)**

A skier traveling 15 m/s reaches the foot of a steady upward 12° incline and glides up along this slope eventually coming to rest.

(a) **How far did he travel along the slope if there was no friction?**

**ANSWER:** Energy at bottom:

\[ E_1 = \frac{1}{2}mv^2 \]

Energy at the end:

\[ E_2 = mgh \]

If there is no friction, \( E_1 = E_2 \), so

\[ h = \frac{v^2}{2g} = \frac{15^2}{2 \times 9.8} \text{ m} = 11.48 \text{ m} \]

The distance he travels is

\[ d = \frac{h}{\sin 12^\circ} = \frac{11.48}{\sin 12^\circ} = 55.22 \text{ m} \]
(b) How far did he travel if there was friction and the coefficient of friction was \( \mu = 0.3 \)?

ANSWER: With friction, energy is lost to heat,

\[
Q = Fd
\]

where \( F \) is friction. We know \( F = \mu N \) and \( N = W \cos 12^\circ \), so

\[
Q = \mu mgd \cos 12^\circ
\]

Conservation of energy:

\[
E_1 = E_2 + Q
\]

so

\[
\frac{1}{2}mv^2 = mgh + \mu mgd \cos 12^\circ
\]

Dividing by \( m \) and using \( h = d \sin 12^\circ \), we obtain

\[
\frac{1}{2}v^2 = gd \sin 12^\circ + \mu gd \cos 12^\circ
\]

Solving for \( d \),

\[
d = \frac{\frac{1}{2}v^2}{g \sin 12^\circ + \mu g \cos 12^\circ} = \frac{\frac{1}{2} \times 15^2}{9.8(\sin 12^\circ + 0.3 \cos 12^\circ)} = 22.9 \text{ m}
\]

(c) How much heat was generated in the two cases above if the mass of the skier (including equipment) is 100 kg?

ANSWER: In case (a) (no friction), there is no heat generated.

In case (b), the heat generated is

\[
Q = \mu mgd \cos 12^\circ = 0.3 \times 100 \times 9.8 \times 22.9 \times \cos 12^\circ = 6585 \ J
\]