Chapter 14

Strings at Strong Coupling

Following *"String Theory"* by J. Polchinski, Vol.II.
Notes written by students (work still in progress).
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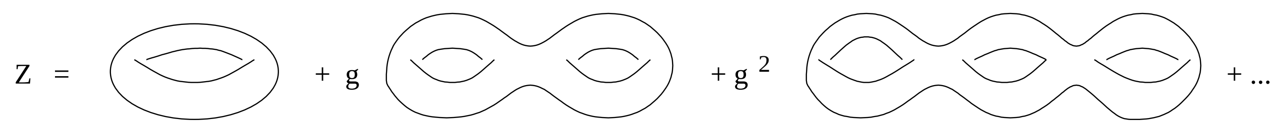


g is a dimensionless coupling constant where $\alpha' = l_s^2(\text{length}^2)$. This series is valid for smalll g and breaks down when g is large, eg. black holes or low energy case. Before considers strings at strong coupling let look at a simple black hole, Schwarzschild black hole.

Schwarzschild black hole

$$ds^2 = -\left(1 - \frac{r_0}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{r_0}{r}} + r^2 \left(d\theta^2 + \sin^2\theta\phi^2\right) \tag{14.1}$$

The horizon is $r_0 = 2GM_{\text{b.h.}}$, which's obtained from $-g_{tt} = 1 - r_0/r$ by letting $r \to \infty$ and expanding in term of $1/r$. The first order is $2V(r)$, where $V(r) = GM/r$. The black hole radiates giving a Hawking temperature, ie.

$$T_H = \frac{\hbar}{4\pi} h'(r_0) = \frac{1}{4\pi r_0}, \tag{14.2}$$

where $h(r) = 1 - r_0/r$ and set $\hbar = 1$. The entropy is

$$S = \frac{A_H}{4G},\ A_H = \text{area of horizon.} \tag{14.3}$$

From the second law of thermodymaics

$$dU = TdS \tag{14.4}$$

$$\begin{aligned} T_H dS_{\text{b.h.}} &= \frac{1}{4\pi r_H} d\left(\frac{4\pi r_H^2}{4G}\right) \\ &= d\left(\frac{r_H}{2G}\right) = dM_{\text{b.h.}} \end{aligned} \tag{14.5}$$

which is agree with the second law. From classical thermodymaics a sphere will radiate with energy density, u

$$\begin{aligned} u &\propto T^4 \\ U &= uV \propto T^4 R^3. \end{aligned}$$

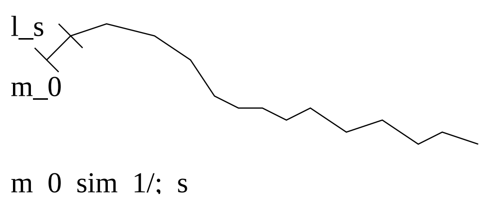
and from schwarzschild black hole

$$\begin{aligned} r_0 &\propto M_{\text{b.h.}} = U \propto T^4 R^3 \text{ but} \\ T &\propto \frac{1}{\sqrt{r_0}} \\ S &\sim \frac{U}{T} \sim T^3 V = r_0^{-3/2} r_0^3 \sim r_0^{3/2} \sim M_{\text{b.h.}}^{3/2} << M_{\text{b.h.}}^2 \end{aligned} \tag{14.6}$$

which is disagree with the second law. Next try to reconcile by considering strings. From Hamiltonian eigenvalue,

$$\begin{aligned} H &\sim \alpha' p^2 + N \\ M^2 &\sim \frac{N}{\alpha'} = \frac{N}{l_s^2},\ \alpha' \ = \ l_s^2. \end{aligned}$$

At high excited strings, $N >> 1$,



The strings looks like ramdom walks with N steps

Strings live in D dimensions, therefore there are D possible directions for strings to move to or $D \times D \times \cdots$, for n steps, D^n possiblities, ie.

$$\begin{aligned} S &\sim \ln D^n = n \ln D \sim n \\ \text{total mass M} &= n m_0 \sim \frac{n}{l_s} \sim \frac{S}{l_s}, \text{ then} \\ S &\sim M l_s \end{aligned} \tag{14.7}$$

but black holes have a strong coulping g or strong gravitational interaction. Therefore let take the strong coupling g into consideration. From classical physics, we know that G is a constant and has unit time^2 and from strings $G \sim g^2\alpha' \sim g^2 l_s^2$(in everyday life experience, curvature $< 1/l_s$ making Einstien's equation still valid). With $G \sim g^2 l_s^2$ constant, there are two possiblities.

(i) If g small, then string perturbation thoery is still valid.

(ii) If g large(l_s small) or $M \sim \frac{\sqrt{N}}{l_s}$ increase, then l_s small enough $l_s \sim r_0$ and $S_{\text{bh.}} \sim \frac{r_0^2}{G} \sim \frac{1}{g^2}$. Then $S \to S_{\text{b.h.}}$ and

$$\begin{aligned} S_{\text{b.h.}}^{1/2} M G^{1/2} &\sim S_{\text{b.h.}} \text{ or} \\ S_{\text{b.h.}} &\sim M^2. \end{aligned} \tag{14.8}$$

Next let consider Kaluza-Klein black hole with the metric and following properties

$$ds^2 = -\frac{1}{\sqrt{f}}(1 - \frac{r_0}{r})dt^2 + \sqrt{f}\frac{dr^2}{1 - \frac{r_0}{r}} + \sqrt{f} r^2 (d\theta^2 + sin^2\theta d\phi^2) \tag{14.9}$$

$$
\begin{aligned}
f(r) &= 1 + \frac{r_0 sinh^2\gamma}{r} && (14.10)\\
h(r) &= \frac{1}{\sqrt{f}}(1 - \frac{r_0}{r})\\
&\approx \left(1 - \frac{r_0 sinh^2\gamma}{2r}\right)(1 - \frac{r_0}{r})\\
&\approx 1 - \frac{r_0}{r}\left(1 + \frac{sinh^2\gamma}{2}\right)\\
2GM &= r_0\left(1 + \frac{sinh^2\gamma}{2}\right) && (14.11)\\
T_H &= \frac{h'(r_0)}{4\pi} = \frac{1}{4\pi r_0\sqrt{f}} = \frac{1}{4\pi r_0 cosh\gamma} && (14.12)\\
A_H &= 4\pi\sqrt{f} r_0^2 = 4\pi r_0^2 cosh\gamma && (14.13)\\
Q &= \frac{r_0}{2G} sinh(2\gamma) && (14.14)\\
n &= \frac{r_0 R}{2G} sinh(2\gamma) && (14.15)
\end{aligned}
$$

where Q, a charge, comes from momentun in the extra dimension and n, a number of state, relates to momentum and the extra dimension radius, R, by

$$
\begin{aligned}
p &= \frac{n}{R}\\
Q &= \frac{p}{2} = \frac{n}{2R}.
\end{aligned}
$$

The 10 dimensional Kaluza-Klein metric, reduced to 4-D, and its properties are

$$
\begin{aligned}
ds^2 &= -h(r)dt^2 + \frac{dr^2}{h(r)} + \sqrt{\Delta} r^2(d\theta^2 + sin^2\theta d\phi^2) && (14.16)\\
h(r) &= \frac{1}{\sqrt{\Delta}}(1 - \frac{r_0}{r})\\
\Delta &= f_1(r) f_2(r) f_3(r) f_4(r) && (14.17)\\
Q_i &= \frac{\lambda_i r_0}{G} sinh(2\gamma_i) && (14.18)\\
M_{\text{b.h.}} &= \frac{r_0}{8G}\prod_{i=1}^{4} cosh(2\gamma_i) && (14.19)\\
S &= \frac{A_H}{4G} = \frac{\pi r_0^2}{G}\prod_{i=1}^{4} cosh\gamma && (14.20)\\
T_H &= \frac{1}{4\pi r_0}\prod_{i=1}^{4}\frac{1}{cosh\gamma_i} && (14.21)\\
dU &= TdS + \sum_i \Phi_i dQ_i. && (14.22)
\end{aligned}
$$

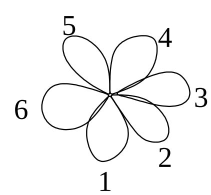
If consider in an extremmal case, with fixed Q_i, $r_0 \to 0$, then $\gamma_i \to \infty$ and

$$
\begin{aligned}
\sinh(2\gamma_i) &\sim \frac{e^{2\gamma_i}}{2}\\
\frac{GQ_i}{\lambda_i} &\sim \frac{r_0}{2} e^{2\gamma_i}
\end{aligned}
$$

$$\begin{aligned}
\sqrt{r_0}e^{\gamma_i} &= \sqrt{\frac{2GQ_i}{\lambda_i}} \\
S &= \frac{\pi r_0^2}{G}\prod_i \frac{e^{\gamma_i}}{2} \\
&= \frac{\pi G}{4}\prod_i \sqrt{\frac{Q_i}{\lambda_i}} && (14.23) \\
M &\sim \frac{1}{8}\sum_{i=1}^{4}\frac{Q_i}{\lambda_i} && (14.24) \\
T_H &= \frac{r_0^2}{4\pi r_0}\prod_{i=1}^{4}\frac{2}{e^{\gamma_i}\sqrt{r_0}}. && (14.25)
\end{aligned}$$

As $r_0 \to 0$, $T_H \to 0$ or no radiation but S still constant, ie. the black hole is still stable(no quatum corrections needed) or no g expansion or this is non-perturbative result.

From string theory, six dimensions are compactified. Let try to do different combinations of D-brane.



R_i, $i = 1$,...,6, compactified dimension index. D6-brane is on $(x^1, .., x^6)$ with Q_1, D2-brane is on (x^1, x^2) with Q_2, and D5-brane is on $(x^2, .., x^6)$ with Q_3, where $p_i = n/R_i$. And count the states on the branes. The entropy and λ_i will be

$$\begin{aligned}
S &= 2\prod_{i=1}^{4}\sqrt{Q_i} && (14.26) \\
\lambda_1 &= \frac{gl_s^7}{8R_1...R_6} \\
\lambda_2 &= \frac{gl_s^3}{8R_1R_2} \\
\lambda_3 &= \frac{g^2l_s^6}{8R_2R_3R_4R_5R_6} \\
\lambda_4 &= \frac{R_2}{8} \\
G &= \frac{g^2l_s^8}{8R_1...R_6}. && (14.27)
\end{aligned}$$

This is agree with the result from GR.