# String Theory II

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# Contents

# UNIT 9

# Low Energy Physics

## 9.1 Type IIA Superstring

Sectors: (Ns+,NS+), (R+,NS+), (NS+,R-), (R+,R-).
(NS+, NS+): massless states: $A_{\mu\nu}\psi^{\mu}_{-1/2}\psi^{\nu}_{-1/2}|0;k\rangle$ $A_{\mu\nu}$ is decomposed into a scalar, antisymmetric field and traceless symmetric field: $8\times 8 = 1+28+35$ The scalar field represents the dilaton. We do not see it in Nature and it is believed to have settled into it's ground state value and not affect dynamics any further. We will set it to zero to avoid complications in an already complicated discussion (it should be set to a constant, but we can always tweak couplings, etc., so setting it to zero will be fine). Let $B_{\mu\nu}$ be the antisymmetric tensor and $g_{\mu\nu}$ the traceless symmetric tensor (graviton). The dynamics of $g_{\mu\nu}$ is described by the Einstein action

$$S_g = \frac{1}{4\pi G_{10}}\int d^{10}x\sqrt{-g}\,R,$$

where $G_{10}$ is the ten-dimensional Newton's constant. This can be derived from string theory tree-level amplitudes and loop amplitudes introduce corrections. $B_{\mu\nu}$ has field strength

$$H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} - \partial_\nu B_{\mu\rho} + \partial_\rho B_{\mu\nu}.$$

$H_{\mu\nu\rho}$ is totally antisymmetric in it's indices. The action is given by

$$S_B = -\frac{1}{8\pi G}\int d^{10}x\sqrt{-g}H^{\mu\nu\rho}H_{\mu\nu\rho},$$

where indices are raised and lowered by $g_{\mu\nu}$.
(R+,R-): massless states: $|\vec{s};k\rangle \otimes |\vec{s'};k\rangle,\;\; 8\times 8$ of them.
Recall $\psi^\mu_0|\vec{s};k\rangle$ is also a ground state (annihilated by all $\psi^\mu_r,\ r>0$).
States decompose into $C_\mu\psi^\mu_0|0\rangle$ and $C_{\mu\nu\rho}\psi^{[\mu,}_0\psi^{[\nu,}_0\psi^{\rho]]}_0|0\rangle$ where we antisymmetrize over all indices. There are 8 $C_\mu$ (transverse $\mu$) and 56 $C_{\mu\nu\rho}$ $(8{+}56 = 64)$

so they span the ground states. The action is given by

$$S_R = -\frac{1}{8\pi G_{10}} \int d^{10}x\sqrt{-g}\left(F_{\mu\nu}F^{\mu\nu} + \tilde{F}_{\mu\nu\rho\sigma}\tilde{F}^{\mu\nu\rho\sigma}\right)$$

where $F_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu$ is the field strength of $C_\mu$.

$$\tilde{F}_{\mu\nu\rho\sigma} = F_{\mu\nu\rho\sigma} - \frac{1}{4}\left(C_\mu H_{\nu\rho\sigma} + C_\nu H_{\rho\sigma\mu} + C_\rho H_{\sigma\mu\nu} + C_\sigma H_{\mu\nu\rho}\right)$$

where $F_{\mu\nu\rho\sigma} = \partial_\mu C_{\nu\rho\sigma} + ...$ (add terms such that $F_{\mu\nu\rho\sigma}$ is completely antisymmetric) and is the field strength of $C_{\mu\nu\rho}$.
There is one more contribution to the action that does not involve the metric (topological). This is a Chern-Simons term given by

$$S_{CS} = -\frac{1}{8\pi G_{10}} \int d^{10}x\epsilon^{\mu_1\mu_2\ldots\mu_{10}} B_{\mu_1\mu_2} F_{\mu_3\mu_4\mu_5\mu_6} F_{\mu_7\mu_8\mu_9\mu_{10}}$$

The total action is the sum of all the actions and is given by

$$S = S_g + S_B + S_R + S_{CS}$$

There is a fermionic counterpart which we will not discuss.

## 9.2 Supergravity

Let us compare with supergravity (SUGRA). Unfortunately, SUGRA lives in 11 dimensions, yet it looks so much like the type-II superstring, that is hard to ignore. It turns out that (modern wisdom holds) we really live in eleven dimensions and ten dimensional strings are really an eleven dimensional theory! What theory? Nobody knows ... M-Theory.
We have seen this problem with dimensions before. We compactified one dimension a la Kaluza-Klein, then let $R \to 0$ and the extra dimension would not go away. Same here. We will compactify the eleventh dimension to get 10D superstrings, but the eleventh dimension will remain lurking in the background.
Action:

$$\begin{aligned} S_{11}^{SUGRA} &= \frac{1}{4\pi G_{11}} \int d^{11}x\sqrt{-G}\left(R^{(11)} - \frac{1}{2}F_{MNQR}F^{MNQR}\right) \\ &\quad -\frac{1}{24\pi G_{11}} \int d^{11}x\ \epsilon^{M_1M_2\ldots M_{11}} A_{M_1M_2M_3} F_{M_4M_5M_6M_7} F_{M_8M_9M_{10}M_{11}} \end{aligned}$$

where $F_{MNQR}$ is the field strength of $A_{MNQ}$ $(F_{MNQR} = \partial_M A_{NQR} + ...)$, where F is completely antisymmetric. The last term is a Chern-Simons term and is gauge invariant

$$\delta A_{MNQ} = \partial_M \lambda_{NQ} + ...$$

even though it doesn't look like it.
We need to reduce the dimension from eleven to ten to compare with superstrings. We will do that a la Kaluza-Klein. Assume nothing depends on the eleventh coordinate and call it "$u$".
The metric: $ds^2 = G_{MN}(x^\mu)dx^\mu dx^\nu, \ \ M,N = 0,1,...,10, \ \mu = 0,1,..,9$ We may decompose the metric as such

$$ds^2 = G_{\mu\nu}dx^\mu dx^\nu + 2G_{\mu u}dx^\mu du + G_{uu}du^2$$

Let $G_u u = 1$ for simplicity (fixes the size of the extra dimension, which can be rescaled later).
Introduce vector $A_m u = G_{\mu u}$ and metric $g_{\mu\nu} = G_{\mu\nu} - A_\mu A_\nu$. Then we may write $ds^2 = g_{\mu\nu}dx^\mu dx^\nu + (du + A_\mu dx^\mu)^2$. The potential $A_{MNQ}$ may also be grouped into $A_{\mu\nu\rho}$ and $A_{\mu\nu} = A_{\mu\nu u}$ (no other components exist, because $A_{MNQ}$ is antisymmetric, so we can not have two $u$ indices). So now the field content becomes (from the 10D perspective) $g_{\mu\nu}, \ A_\mu, \ A_{\mu\nu}, \ A_{\mu\nu\rho}$, very similar to the type-II superstring.
Futhermore,

$$R^{(11)} = R^{(10)} - \frac{1}{2}F_{\mu\nu}F^{\mu\nu}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

$$F_{MNQR}F^{MNQR} = F_{\mu\nu\rho}F^{\mu\nu\rho} + \tilde{F}_{\mu\nu\rho\sigma}\tilde{F}^{\mu\nu\rho\sigma}$$

where $F_{\mu\nu\rho} = \partial_\mu A_{\mu\rho} + ...$ , $\tilde{F}_{\mu\nu\rho\sigma} = F_{\mu\nu\rho\sigma} - A_\mu F_{\nu\rho\sigma} + ...$ and $F_{\mu\nu\rho\sigma} = \partial_\mu A_{\nu\rho\sigma} + ...$ and

$$\begin{aligned} \frac{1}{6}\epsilon^{M_1M_2...M_{11}}A_{M_1M_2M_3}F_{M_4M_5M_6M_7}F_{M_8M_9M_{10}M_{11}} &= \epsilon^{\mu_1\mu_2...\mu_{10}}A_{\mu_1\mu_2}F_{\mu_3\mu_4\mu_5\mu_6}F_{\mu_7\mu_8\mu_9\mu_{10}} \\ &= \epsilon^{\mu_1\mu_2...\mu_{10}}A_{\mu_1\mu_2\mu_3}F_{\mu_4\mu_5\mu_6}F_{\mu_7\mu_8\mu_9\mu_{10}} + \text{total derivative} \end{aligned}$$

The last equality can easily be verified by integrating by parts.
Also $\sqrt{-G} = \sqrt{-g}$.
If the eleventh dimension has length $2\pi R$, then the action becomes

$$\begin{aligned} S_{11}^{SUGRA} &= \frac{1}{4\pi G_{10}}\int d^{10}x\sqrt{-g}\bigg(R^{(10)} - \frac{1}{2}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}F_{\mu\nu\rho}F^{\mu\nu\rho} - \frac{1}{2}\tilde{F}_{\mu\nu\rho\sigma}\tilde{F}^{\mu\nu\rho\sigma} \\ &\quad -\frac{1}{2}\epsilon^{\mu_1\mu_2...\mu_{10}}A_{\mu_1\mu_2}F_{\mu_3\mu_4\mu_5\mu_6}F_{\mu_7\mu_8\mu_9\mu_{10}}\bigg) \end{aligned}$$

where $G_{10} = 2\pi RG_{11}$ is the 10D Newton's constant and we have rescaled $A_{\mu\nu} \to \frac{1}{\sqrt{2\pi R}}A_{\mu\nu}, \ \ A_{\mu\nu\rho} \to \frac{1}{\sqrt{2\pi R}}A_{\mu\nu\rho}$.
This action is identical to the one obtained from type-IIA superstring if we identify

$$g_{\mu\nu} \to g_{\mu\nu}, \ \ A_\mu \to C_\mu, \ \ A_{\mu\nu} \to B_{\mu\nu}, \ \ A_{\mu\nu\rho} \to C_{\mu\nu\rho}.$$