PHYSICS 612 - SPRING 2019

Homework Set 3

due date: Wed., April 3, 2019

Problem 3.1
Pair Annihilation. Consider the process

$$e^+(p_1)e^-(p_2) \to \gamma(\epsilon_1, k_1)\gamma(\epsilon_2, k_2)$$

(a) Draw the two diagrams that contribute at tree level. Write the amplitude in the form

$$i\mathcal{A} = \epsilon_1^\mu \epsilon_2^\nu \mathcal{M}_{\mu\nu}$$

and find an expression for $\mathcal{M}_{\mu\nu}$.

Show that the Ward identity holds:

$$k_1^\mu \mathcal{M}_{\mu\nu} = 0$$

Hence show that summing over photon polarizations yields

$$\sum_{\text{pol}} |\mathcal{A}|^2 = \mathcal{M}^*_{\mu\nu}\mathcal{M}^{\mu\nu}$$

(b) After averaging over spins, show that the amplitude may be expressed in terms of p_1, k_1, k_2 as

$$\langle|\mathcal{A}|^2\rangle = 2e^4 \left\{ \frac{p_1 \cdot k_2}{p_1 \cdot k_1} + \frac{p_1 \cdot k_1}{p_1 \cdot k_2} + 2m^2 \left(\frac{1}{p_1 \cdot k_1} + \frac{1}{p_1 \cdot k_2} \right) - m^4 \left(\frac{1}{p_1 \cdot k_1} + \frac{1}{p_1 \cdot k_2} \right)^2 \right\}$$

(c) Using the formula for the differential cross section derived in class

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{k_{final}}{k_{initial}} \langle|\mathcal{A}|^2\rangle$$

show that in the center-of-mass (lab) frame

$$\frac{d\sigma}{d\cos\theta} = \frac{e^4}{32\pi\beta E^2} \left\{ \frac{1+\beta^2\cos^2\theta}{1-\beta^2\cos^2\theta} + \frac{2(1-\beta^2)}{1-\beta^2\cos^2\theta} - \frac{2(1-\beta^2)^2}{(1-\beta^2\cos^2\theta)^2} \right\}$$

where E is the energy of the electron and β is its speed.

Problem 3.2

Bhabha scattering. Consider the process

$$e^+e^- \to e^+e^-$$

in the high energy limit in which you may ignore the mass of the electron.

(a) Draw the two diagrams that contribute at tree level.

Obtain an expression of the amplitude.

(b) After averaging and summing over spins, show that the amplitude may be written as

$$\langle|\mathcal{A}|^2\rangle = \frac{1}{4}\sum_{\text{spins}}|\mathcal{A}|^2 = 2e^4\left\{u^2\left(\frac{1}{s}+\frac{1}{t}\right)^2 + \left(\frac{t}{s}\right)^2 + \left(\frac{s}{t}\right)^2\right\}$$

in terms of the Mandelstam variables s, t, u.

(c) Using the formula for the cross section in **Problem 3.1*(c)***, find an expression for the cross section in terms of $\cos\theta$.

Problem 3.3

Exotic contributions to the anomalous magnetic moment. The Standard Model of particle interactions predicts the existence of a scalar ϕ (the Higgs boson) which couples to the electron. The Lagrangian density to be added is

$$\mathcal{L}_\phi = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m_H^2\phi^2 - \frac{\lambda}{\sqrt{2}}\phi\bar{\psi}\psi$$

(a) Deduce the Feynman rules for the Higgs boson. Draw the one-loop diagram that will contribute to the anomalous magnetic moment of the electron and write down an expression for it.

(b) Isolate the part of the diagram which is proportional to the magnetic moment and calculate its contribution to the g-factor. (Recall that $g = 2(1 + \frac{e^2}{8\pi^2} + \ldots)$ in QED.)

(c) Let g_{exp} be the experimental value of g. The error is

$$|g_{\text{exp}} - g| < 2 \times 10^{-10}$$

What limits does this place on λ and m_H?