Quantum Field Theory II

GEORGE SIOPSIS

Department of Physics and Astronomy
The University of Tennessee
Knoxville, TN 37996-1200
U.S.A.
e-mail: `siopsis@tennessee.edu`

Spring 2007

Contents

UNIT 1

Interactions

1.1 Scattering theory and the S-matrix

Until now, we have only considered free fields described by a Hamiltonian H_0, say, which we were able to diagonalize (found all eigenvalues and corresponding eigenstates). We studied *kinematics*. To describe Nature, we need to include interactions and study *dynamics*. This is usually a much harder problem. The Hamiltonian is modified to

$$H = H_0 + H_I \tag{1.1.1}$$

It is no longer possible to solve the eigenvalue problem except in very few special cases (mostly in two spacetime dimensions). One usually resorts to perturbation theory assuming H_I is small. This does not answer deep questions, such as *"what is a proton?"* but does provide a method for some very accurate calculations (e.g., the magnetic moment of the electron is known to about 10 significant figures both theoretically and experimentally and they agree!)

Experimental results are obtained primarily through scattering: two beams collide and the products are observed at the detector. To compare with them, we need to develop *scattering theory*. It turns out that scattering processes hold *all* the information of quantum field theory.

In quantum mechanics, H_I is usually represented by a potential. If it has compact support, then at times $t \to \pm\infty$, $H = H_0$. Thus, we may define incoming and outgoing states which are eigenfunctions of H_0. We shall attempt to do the same in quantum field theory.

Consider a state $|\mathrm{in}\rangle$ in the Hilbert space of the full Hamiltonian H. It evolves in time as

$$e^{-iHt}|\mathrm{in}\rangle \tag{1.1.2}$$

As $t \to -\infty$, we expect $H \to H_0$ (no interactions), so asymptotically, our state approaches a state in the Hilbert space of H_0. Call that state $|\mathrm{in}, 0\rangle$. It evolves in time as

$$e^{-iH_0 t}|\mathrm{in}, 0\rangle \tag{1.1.3}$$

The statement that this is asymptotic to $|\mathrm{in}\rangle$ then amounts to

$$|\mathrm{in}\rangle = \lim_{t\to-\infty} U^\dagger(t)|\mathrm{in}, 0\rangle \ , \quad U(t) \equiv e^{+iH_0 t}e^{-iHt} \tag{1.1.4}$$

The operator $U(t)$ maps a state in the Hilbert space of H to a state in the Hilbert space of H_0. It is a unitary operator (provided H, H_0 are Hermitian). If H and H_0 commute, then we may write $U(t) = e^{-iH_I t}$. This is rarely the case. In general, $U(t)$ is a very complicated object.

Similarly, in the infinite future we may map

$$|\mathrm{out}\rangle = \lim_{t\to+\infty} U^\dagger(t)|\mathrm{out}, 0\rangle \tag{1.1.5}$$

where $|\mathrm{out}\rangle$ ($|\mathrm{out}, 0\rangle$) is in the Hilbert space of H (H_0).

We wish to calculate the amplitude of the general process

$$|\mathrm{in}\rangle \to |\mathrm{out}\rangle \tag{1.1.6}$$

and compare our results with experiment (experimentalists measure *probabilities*, i.e., $|\langle out|in\rangle|^2$). The amplitude can be written as

$$\langle\mathrm{out}|\mathrm{in}\rangle = \langle\mathrm{out}, 0|S|\mathrm{in}, 0\rangle \ , \quad S \equiv \lim_{t_\pm\to\pm\infty} U(t_+)U^\dagger(t_-) \tag{1.1.7}$$

where the S-matrix maps in-asymptotes to out-asymptotes. These two Hilbert spaces may in general be distinct, but we shall not deal with such cases here.

S contains *all* the information of quantum field theory. It is an important object to study.

1.2 Wick's theorem

1.3 Feynman diagrams

1.4 Counterterms

1.5 Reaction rates

1.6 Crossing symmetry and relations

1.7 Unitarity

1.8 Dispersion relations

1.9 Statistical mechanics

1.10 Renormalization

1.11 Higher-order radiative corrections

1.12 Green functions

1.13 Vacuum expectation values

PROBLEMS

UNIT 2

Path Integrals

UNIT 3

Quantum Electrodynamics

UNIT 4

The Standard Model

UNIT 5

And Beyond