PHYSICS 612 - SPRING 2007

Homework Set 1

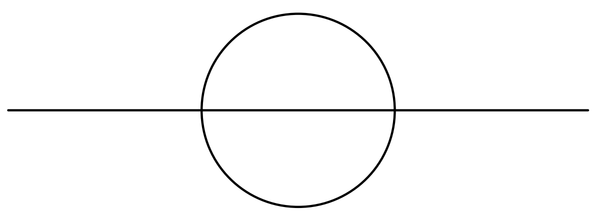
due date: March 21, 2007

Problem 1.1
Consider the 3-body decay of a particle of mass M decaying into two massless particles and one massive particle of mass m ($M > m$). Find the decay rate Γ in terms of the amplitude $\mathcal{A}$. What is the available phase space? Draw the allowed region in the plane with axes E_1 and E_2, the energies of the two massless particles.

Problem 1.2
In the ϕ^3 theory draw the three-point diagram at order λ^3. Calculate its amplitude $\mathcal{A}(s,t,u)$, where $s = p_1^2$, $t = p_2^2$, $u = p_3^2$, and p_i^μ ($i = 1,2,3$) are the (not necessarily physical) momenta of the three external legs. Show that the signularities agree with the predictions from the unitarity theorem. You will find the following Feynman trick useful:

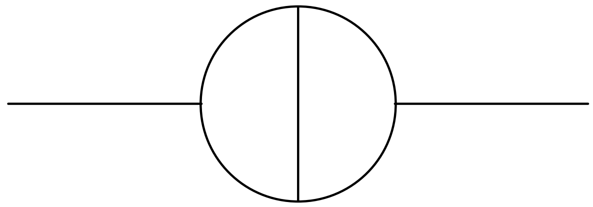
$$\frac{1}{\alpha\beta\gamma} = \int_0^1 dx \int_0^{1-x} dy \frac{1}{(x\alpha + y\beta + (1-x-y)\gamma)^3}$$

Problem 1.3
The order λ^2 contribution to the propagator in the ϕ^4 theory is



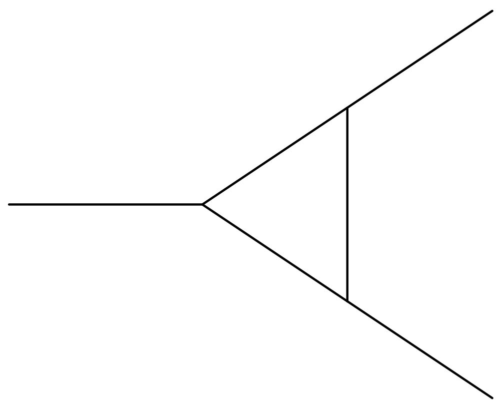
Show that it is quadratically divergent, of the form $C\lambda^2\Lambda^2$, and compute C. Show that this generates mass renormalization.
[HINT: rescale the momenta after a Wick rotation by Λ and then expand the integrand in $1/\Lambda$.]

Problem 1.4
Show that the divergence of the $o(\lambda^2)$ diagram

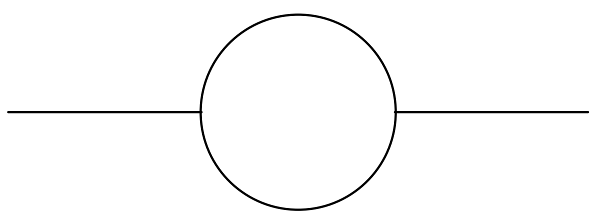


in ϕ^3 theory is canceled by the $o(\lambda)$ counterterms.

Problem 1.5
Consider the ϕ^3 theory in <u>six</u> space-time dimensions. Show that the $o(\lambda^3)$ three-point diagram



is logarithmically divergent (i.e., of the form $C\lambda^3 \ln \Lambda + \ldots$) and the $o(\lambda^2)$ propagator



is quadratically divergent (i.e., of the form $C'\lambda^2\Lambda^2 + \ldots$).
In each case find the coefficient of the leading divergence.
Is this theory super-renormalizable, renormalizable or non-renormalizable? Explain.