PHYSICS 611 - FALL 2018

Homework Set 6 (Extra Credit)

due date: Fri., December 7, 2018

Show all your work for full credit.

Problem 6.1

Derive the equations of motion (Maxwell equations) for the non-abelian gauge theory with Lagrangian density

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} - J^{a\mu} A^a_\mu$$

By demanding gauge invariance, obtain the continuity equation for the current J^a_μ, and show that it is not conserved.

Problem 6.2

If ψ represents a quark, show that a meson (quark-antiquark pair)

$$\psi^{(1)\dagger}\psi^{(2)}$$

and a baryon (3-quark particle)

$$\epsilon^{abc}\psi^{(1)a}\psi^{(2)b}\psi^{(3)c} = \begin{vmatrix} \psi^{(1)1} & \psi^{(2)1} & \psi^{(3)1} \\ \psi^{(1)2} & \psi^{(2)2} & \psi^{(3)2} \\ \psi^{(1)3} & \psi^{(2)3} & \psi^{(3)3} \end{vmatrix}$$

are gauge-invariant.

Problem 6.3

From the Lagrangian density for the Higgs,

$$\mathcal{L}_{Higgs} = (D_\mu\phi)^\dagger D_\mu\phi - \lambda\left(\phi^\dagger\phi - \frac{v^2}{2}\right)$$

where $D_\mu = \partial_\mu + igW^a_\mu \frac{\sigma^a}{2} + g' B_\mu Y$, obtain the masses of the photon, W and Z bosons, and the Higgs particle, respectively,

$$m_\gamma = 0\ , \quad m_W = \frac{ev}{2\sin\theta_W}\ , \quad m_Z = \frac{ev}{2\sin\theta_W\cos\theta_W}\ , \quad m_{Higgs} = \sqrt{2\lambda}v$$

Problem 6.4

Show that the Yukawa Lagrangian density for the electron,

$$\mathcal{L}_{Yukawa} = -f_e(\bar{\nu}_{eL} \ \ \bar{e}_L)\phi^C e_R + h.c.$$

is gauge-invariant, and the mass of the electron is $m_e = f_e v/\sqrt{2}$.
What term would you add to the Lagrangian density to give the neutrino, ν_e, mass?

Problem 6.5
Show that the Lagrangian density for the left-handed quarks,

$$\mathcal{L}_{L,quark} = (\bar{u}_L \;\; \bar{d}_L) i\gamma^\mu D_\mu \begin{pmatrix} u_L \\ d'_L \end{pmatrix} + (\bar{c}_L \;\; \bar{s}_L) i\gamma^\mu D_\mu \begin{pmatrix} c_L \\ s'_L \end{pmatrix} + (\bar{t}_L \;\; \bar{b}_L) i\gamma^\mu D_\mu \begin{pmatrix} t_L \\ b'_L \end{pmatrix}$$

where

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = U \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

and U is a 3×3 unitary matrix (Kobayashi-Maskawa matrix), is gauge-invariant.