PHYSICS 611 - FALL 2018
Homework Set 2

due date: Wed., September 26, 2018

Show all your work for full credit.

Problem 2.1

Find the Lorentz matrix Λ for a boost with velocity v in the x-direction and show that for $|v| \ll 1$,

$$\phi(\Lambda x) = \phi(x) - v(t\partial_x + x\partial_t)\phi$$

Show that the operator generating these boosts is

$$M_x = -P_x t - \int d^3x \; x \, \mathcal{H}$$

i.e.,

$$[M_x, \phi] = -i(t\partial_x + x\partial_t)\phi$$

For finite velocities, show that this exponentiates to

$$U(\Lambda)\phi(x)U^{-1}(\Lambda) = \phi(\Lambda x) \;\;,\;\; U(\Lambda) = e^{i\zeta M_x}$$

where ζ is the rapidity.

Problem 2.2

Show that the charge conjugation operator U_C commutes with the Hamiltonian and anti-commutes with the charge,

$$[U_C, H] = 0 \;\;,\;\; \{U_C, Q\} = 0$$

What does this imply about the action of charge conjugation on the eigenstates of H and Q?

Problem 2.3

Find the Noether currents for the three boosts and the three rotations and calculate the commutators of the corresponding Noether charges.

Problem 2.4

Consider two complex scalar fields, ϕ_a ($a = 1, 2$), each of mass m.

(a) Show that there are four conserved charges,

$$Q^\mu = \frac{i}{2} \int d^3x \; (\phi_a^*(\sigma^\mu)_{ab}\pi_b^* - \pi_a(\sigma^\mu)_{ab}\phi_b)$$

where σ^0 is the identity 2×2 matrix and σ^i are the three Pauli matrices. Find the transformation generated by each of the four charges and calculate the corresponding Noether currents.

(b) Show that the three charges Q^i obey the commutation relations of angular momentum.

(c) This system possesses two additional symmetries (total of six). Can you find them? What algebra do their commutators form?
[Hint: think of the fields ϕ_a as <u>four</u> free real fields.]