PHYSICS 611 - FALL 2018

Homework Set 1

due date: Fri., September 7, 2018

Problem 1.1

Using

$$\mathcal{H} = \frac{1}{2}\pi^2 + \frac{1}{2}(\vec{\nabla}\phi)^2 + \frac{1}{2}m^2\phi^2 \ , \ \ \vec{\mathcal{P}} = -\pi\vec{\nabla}\phi$$

express the total four-momentum

$$P_\mu = \int d^3x \mathcal{P}_\mu \ , \ \ \mathcal{P}_\mu = (\mathcal{H}, \vec{\mathcal{P}})$$

in terms of creation and annihilation operators.

Problem 1.2

Show that the measure

$$d^3\Sigma_k = \frac{d^3k}{(2\pi)^3 2\omega_k}$$

is invariant under a boost in the x-direction.
Explain (without a calculation) why $d^3\Sigma_k$ is invariant under rotations.

Problem 1.3

Calculate the inner product

$$\langle \vec{x}, t | \vec{p} \rangle$$

where $|\vec{x}, t\rangle = \phi(\vec{x}, t)|0\rangle$.

Problem 1.4

Show that the transformation property of creation operators under spacetime translations,

$$e^{iP\cdot\lambda} a^\dagger(\vec{p}) e^{-iP\cdot\lambda} = e^{ip\cdot\lambda} a^\dagger(\vec{p})$$

(where P_μ is the total four-momentum) implies the transformation law

$$e^{iP\cdot\lambda} \phi(x) e^{-iP\cdot\lambda} = \phi(x + \lambda)$$

Problem 1.5

Show that the state

$$|\vec{p}_1, \ldots, \vec{p}_N\rangle = C a^\dagger(\vec{p}_1) \cdots a^\dagger(\vec{p}_N)|0\rangle$$

is an eigenstate of the Hamiltonian H and total momentum $\vec{P}$ and find the corresponding eigenvalues.