Problem 1.1
Using
\[ H = \frac{1}{2} \pi^2 + \frac{1}{2} (\vec{\nabla} \phi)^2 + \frac{1}{2} m^2 \phi^2, \quad \vec{P} = -\pi \vec{\nabla} \phi \]
express the total four-momentum
\[ P_\mu = \int d^3x P_\mu, \quad P_\mu = (H, \vec{P}) \]
in terms of creation and annihilation operators.

Problem 1.2
Show that the measure
\[ d^3 \Sigma_k = \frac{d^3 k}{(2\pi)^3 2\omega_k} \]
is invariant under a boost in the \( x \)-direction.
Explain (without a calculation) why \( d^3 \Sigma_k \) is invariant under rotations.

Problem 1.3
Calculate the inner product
\[ \langle \vec{x}, t | \vec{p} \rangle \]
where \( |\vec{x}, t \rangle = \phi(\vec{x}, t)|0\rangle \).

Problem 1.4
Show that the transformation property of creation operators under spacetime translations,
\[ e^{iP \cdot \lambda} a^\dagger(\vec{p}) e^{-iP \cdot \lambda} = e^{iP \cdot \lambda} a^\dagger(\vec{p}) \]
(where \( P_\mu \) is the total four-momentum) implies the transformation law
\[ e^{iP \cdot \lambda} \phi(x) e^{-iP \cdot \lambda} = \phi(x + \lambda) \]

Problem 1.5
Show that the state
\[ |\vec{p}_1, \ldots, \vec{p}_N \rangle = Ca^\dagger(\vec{p}_1) \cdots a^\dagger(\vec{p}_N)|0\rangle \]
is an eigenstate of the Hamiltonian \( H \) and total momentum \( \vec{P} \) and find the corresponding eigenvalues.