Ginzburg-Landau Theory of Phase Transitions

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Dec 5, 2008
Phase Transitions

Ginzburg-Landau Theory

Ginzburg-Landau Theory of Ferromagnetism

Ginzburg-Landau Theory of Superconductivity

Summary
Phase Transitions

- Occurs when a system changes its phase.
- E.g. ice to water, paramagnet to ferromagnet, metal to superconductor
- First order (crystal to liquid) and second order (paramagnet to ferromagnet)
- Characterized by a order parameter.
How to skin this rabbit?

- To study a system, we need to calculate the partition function $Z = \text{tr} \exp(-\mathcal{H}/kT)$.

- Then we can calculate other properties. E.g. $M = kT \frac{\partial Z}{\partial B}$.

- But finite sum of $\exp(-\mathcal{H}/kT)$ is smooth.

- Calculating infinite sum is impossible in most systems.
Ginzburg’s and Landau’s idea

- Expand the free energy in terms of the order parameter.
- Take into account the symmetry.
- For homogenous system:
  \[ F(T) = F_0 + a(T)|\psi|^2 + \frac{1}{2} b(T)|\psi|^4 + \cdots \]
- For inhomogeneous add \( \frac{\hbar^2}{2m} |\nabla \psi|^2 \).
- Under magnetic field add \( \frac{\hbar^2}{2m} \left| (\frac{1}{i}\nabla + 2eA) \psi \right|^2 \).
Ginzburg-Landau Theory

- Truncate after the third term.
- We must have $b > 0$, otherwise no minimum.
- For $a > 0$ we have one minimum.
- For $a < 0$ we have two minimum, whenever $|\psi|^2 = -a(T)/b(T)$. 
Ginzburg-Landau Theory of Ferromagnetism

- Express the free energy in terms of magnetization $M$.
  $$F = F_0 + a(T)M^2 + \frac{1}{2}b(T)M^4.$$  
- Let $b$ be a constant and $a(T) = a_0(T - T_c)$.
- After minimizing with respect to $M$, we get the condition
  $$2M[a_0(T - T_c) + bM^2] = 0.$$  
- This implies either $M = 0$ or $M = \pm\left[\frac{a_0(T_c - T)}{b}\right]^{1/2}$.
- The second condition is valid only when $T < T_c$.  

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Ginzburg-Landau Theory of Phase Transitions
Assume that a superconducting state is characterized by a complex order parameter \( \psi = |\psi| e^{i\theta} \).

\[
f_s(T) = f_n(T) + a_0(T - T_c)|\psi|^2 + \frac{b}{2}|\psi|^4.
\]

Then everything is similar to the theory of ferromagnetism.

We can also calculate the critical field:

\[
f_s(T) - f_n(T) = -\frac{a_0^2(T - T_c)^2}{2b} = -\mu_0 \frac{H_c^2}{2}.
\]

\[
H_c = \frac{a_0}{(\mu_0 b)^{\frac{1}{2}}}(T_c - T).
\]
Inhomogeneous Superconductors

- Free energy density:
  \[ f_s(T) = f_n(T) + \frac{\hbar^2}{2m} |\nabla \psi|^2 + a(T) |\psi|^2 + \frac{b}{2} |\psi|^4. \]

- Total energy:
  \[ F_s(T) = F_n(T) + \int \left( \frac{\hbar^2}{2m} |\nabla \psi|^2 + a(T) |\psi|^2 + \frac{b}{2} |\psi|^4 \right) d^3r. \]

- After minimization with respect to \( \psi \):
  \[ -\frac{\hbar^2}{2m} \nabla^2 \psi + a \psi + b |\psi|^2 = 0. \]

- Solution is:
  \[ \psi(x) = \psi_0 \tanh \left( \frac{x}{\sqrt{2} \xi(T)} \right). \]

- Coherence length \( \xi = \left( \frac{\hbar^2}{2m |a(T)|} \right)^{\frac{1}{2}}. \)
Phase transition occurs when a system changes its phase.

Ginzburg and Landau had a brilliant idea to express free energy in terms of order parameter.

We can study various properties of the system by minimizing the free energy and solving the resulting equation.

Ginzburg-Landau theory does not give a microscopic explanation for phase transition.