PHYSICS 522 - SPRING 2011

Midterm Exam II

Problem 1

Consider a system consisting of a spin 1/2 particle and a spin 3/2 particle governed by the Hamiltonian

$$H = a\vec{S}_1 \cdot \vec{S}_2$$

where $\vec{S}_1$ and $\vec{S}_2$ are the two spin operators.

(a) Find the energy levels of the system and their degeneracies.

(b) Express the eigenvectors of H in terms of the common eigenvectors of $\{\vec{S}_1^2, S_{1z}, \vec{S}_2^2, S_{2z}\}$.

Problem 2

Consider a two-level system governed by the Hamiltonian

$$H_0 = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}$$

where $E_1 < E_2$.
Apply a perturbation λW, where

$$W = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

The Hamiltonian of the system is now

$$H = H_0 + \lambda W$$

Assume $\lambda \ll E_2 - E_1$.

(a) Find the exact energy levels of the perturbed system (eigenvalues of H) and corresponding eigenvectors.

(b) Use second order perturbation theory to calculate the energy levels to second order in λ and corresponding eigenvectors to first order in λ.

Compare your results to the exact expressions obtained in part *(a)*.

Problem 3

A particle of mass m is moving in the x-direction under the influence of the potential

$$V(x) = g|x| \quad , \qquad g > 0$$

Estimate the ground state energy E_0 by using the variational method with the trial function

$$\phi_\alpha(x) = \begin{cases} \alpha - |x| & , \quad |x| < \alpha \\ 0 & , \quad |x| > \alpha \end{cases}$$

Compare your result with the exact value

$$E_0 = a \left(\frac{\hbar^2 g^2}{2m} \right)^{1/3} \quad , \qquad a = 1.019\ldots$$

[CAUTION: $\phi''_\alpha(x)$ is not defined when $\phi'_\alpha(x)$ is discontinuous. Integrate by parts to get rid of second derivatives before you evaluate any integrals.]

Problem 4

An electron-positron pair is created. They are both spin 1/2 particles. Suppose that the system has total spin $S = 0$ and the two particles travel in opposite directions. Observer A measures the spin of the electron whereas observer B measures the spin of the positron.

(a) What is the state of the system?

(b) If B makes no measurement, <u>calculate</u> the probability that A will find the spin of the electron to be pointing in the positive z-direction.

(c) If B makes a measurement and finds that the spin of the positron is in the direction of the unit vector

$$\hat{n} = \begin{pmatrix} \sin\theta \\ 0 \\ \cos\theta \end{pmatrix}$$

<u>calculate</u> the probability that A will find the spin of the electron to be in the positive z-direction.