PHYSICS 522 - SPRING 2011

Midterm Exam I

Problem 1

An incoming beam of particles represented by the wavefunction

$$\varphi_{in}(\vec{r}) = e^{ikz}$$

gives rise to a scattered (outgoing) wave represented by

$$\varphi_S(\vec{r}) = f(\theta, \phi)\frac{e^{ikr}}{r}$$

Find the probability current $\vec{J}_S$ corresponding to the scattered wave $\varphi_S(\vec{r})$ and show that the total probability current

$$I_S = \int \vec{J}_S \cdot d\vec{S} \ ,$$

where the integral is over a sphere of radius R, approaches a constant $\mathcal{C}$ as $R \to \infty$. What is $\mathcal{C}$ and how is it related to the total cross section?

You may use without proof:

$$\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}$$

Problem 2

Consider the square well potential

$$V(r) = \begin{cases} V_0 & , \quad r < r_0 \\ 0 & , \quad r > r_0 \end{cases}$$

(a) Calculate the Born scattering amplitude $f_B(\theta)$.

(b) Show that in the low energy limit, the differential Born cross section is isotropic and the total Born cross section is given by

$$\sigma_B = \frac{16\pi\mu^2 V_0^2 r_0^6}{9\hbar^4}$$

You may use without proof:

$$\cos x = 1 - \frac{1}{2}x^2 + \dots \quad , \qquad \sin x = x - \frac{1}{6}x^3 + \dots$$

Problem 3

Consider the potential

$$V(r) = \begin{cases} \infty & , \quad r < a \\ -V_0 & , \quad a < r < b \\ 0 & , \quad r > b \end{cases}$$

where $V_0 > 0$.

(a) Find an expression for the s-wave phase shift δ_0.

(b) Estimate the total cross section in the low energy limit if you are not near a resonance.

(c) For what values of V_0 do resonances occur?

Problem 4

A beam of particles is incident on a lattice which can be represented by a potential $V(\vec{r})$ with the translation invariance property

$$V(\vec{r} + \vec{R}) = V(\vec{r})$$

for a fixed vector $\vec{R}$.
Let $\vec{k}_i$, $\vec{k}_S$ be the wavevectors of the incoming and scattered particles, respectively.
Show that the Born scattering amplitude $f_B(\theta, \phi)$ vanishes, <u>unless</u> the momentum transfer $\hbar\vec{q}$, where

$$\vec{q} = \vec{k}_i - \vec{k}_S \ ,$$

is a reciprocal lattice vector, i.e., it satisfies the *Laue* condition

$$\vec{q} \cdot \vec{R} = 2\pi n$$

where n is an integer.
[HINT: Shift the vector you integrate over by $\vec{R}$.]