Problem 1

(a) By integrating over the angles, show that for a central potential $V(r)$ the Born scattering amplitude is given by

$$f_B(\theta) = -\frac{2\mu}{\hbar^2} \int_0^\infty dr \frac{r \sin(Kr)}{K} V(r), \quad K^2 = 2k^2(1 - \cos\theta)$$

(b) Calculate the Born scattering amplitude for the square well potential

$$V(r) = \begin{cases} V_0, & r < r_0 \\ 0, & r > r_0 \end{cases}$$

[Hint: Integrate by parts.]

(c) Show that in the low energy limit ($kr_0 \ll 1$), the differential Born cross section is isotropic and the total Born cross section is given by

$$\sigma_B = \frac{16\pi\mu^2V_0^2r_0^6}{9\hbar^4}$$

You may use the expansions \( \cos x = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + \ldots \), \( \sin x = x - \frac{1}{6}x^3 + \frac{1}{120}x^5 + \ldots \)

Problem 2

Consider a system consisting of a spin-1/2 particle and a spin-3/2 particle. Label their respective spins as $\vec{S}_1$ and $\vec{S}_2$ and let $\vec{S} = \vec{S}_1 + \vec{S}_2$ be the total angular momentum of the system.

(a) Write down the eigenstates of the set $\{S_1^2, S_2^2, S_1z, S_2z\}$.

Express the eigenstates of the set $\{S^2, S_z\}$ in terms of the eigenstates of the former set.

(b) Deduce all the Clebsch-Gordan coefficients.

Problem 3

The magnetic moment operator for the proton is

$$\vec{M} = \frac{e}{2m_p c}(\vec{L} + g_S \vec{S}) \quad , \quad g_S = 5.587$$

where $e$ is its charge and $m_p$ is its mass.

Let $\vec{J} = \vec{L} + \vec{S}$ be the total angular momentum. Denote by $\ell$ and $j$ the quantum numbers corresponding to the operators $L^2$ and $J^2$.

(a) For a given $\ell$, what are the possible values of $j$?

(b) For given $j$ and $\ell$, show that the magnetic moment can be written as

$$\vec{M} = \frac{e}{2m_p c} g_J \vec{J}$$

and find $g_J$. 