PHYSICS 522 - SPRING 2011

Homework Set 1

due date: Tue., February 1, 2011

Problem 1.1
Show that, when a particle of mass $m_1$ collides elastically with a particle of mass $m_2$ that is initially at rest, all the recoil (mass $m_2$) particles are scattered in the forward hemisphere ($\theta < \frac{\pi}{2}$).

Problem 1.2
Consider the potential

$$V(r) = \begin{cases} V_0, & r < r_0 \\ 0, & r > r_0 \end{cases}$$

Using the method of partial waves, show that for

$$|V_0| \ll E, \quad kr_0 \ll 1$$

the differential cross section is isotropic and the total cross section is given by

$$\sigma = \frac{16\pi \mu^2 V_0^2 r_0^6}{9\hbar^4}$$

Problem 1.3
Compute and make a polar plot of the differential scattering cross section for a perfectly rigid sphere of radius $r_0$ when

$$kr_0 = \frac{1}{2}$$

using the first three partial waves ($\ell = 0, 1, 2$).

Compute the total cross section using these three terms.

Problem 1.4
Consider scattering by a repulsive $\delta$-shell potential

$$V(r) = \frac{\gamma \hbar^2}{2\mu} \delta(r - r_0)$$

where $\gamma > 0$.

Set up an equation that determines the $s$-wave phase shift $\delta_0$ as a function of $k$.

Show that for large $\gamma$ your result resembles the corresponding result for a hard sphere as long as $\tan(kr_0)$ is not close to zero.

What happens when $\tan(kr_0)$ is close to zero?
Problem 1.5

Using the Born approximation for a Yukawa potential

\[ V(r) = V_0 \frac{e^{-\alpha r}}{r} \]

calculate the first three phase shifts \( \delta_\ell (\ell = 0, 1, 2) \) assuming they are small \( (|\delta_\ell| \ll 1) \).

When the de Broglie wavelength is much longer than the range of the potential, show that \( \delta_\ell \) is proportional to \( k^{2\ell+1} \) and find the proportionality constant in each of the three cases \( \ell = 0, 1, 2 \).

Problem 1.6

Calculate the elastic scattering and absorption cross sections for a potential of the form

\[ V(r) = \begin{cases} 
-V_0(1 + i\xi) & , \quad r < r_0 \\
0 & , \quad r > r_0
\end{cases} \]

where \( V_0, \xi > 0 \), in the low energy limit.

You may assume that \( \xi \ll 1 \) and there is neither a resonance nor a Ramsauer-Townsend effect.

Problem 1.7

Using the optical theorem or otherwise, show that the differential elastic cross section in the forward direction obeys the inequality

\[ \left. \frac{d\sigma}{d\Omega} \right|_{\theta=0} \geq \left( \frac{k\sigma}{4\pi} \right)^2 \]

where \( \sigma \) is the total cross section.