Problem 1
The magnetic moment of a nucleon is
\[ \vec{\mu}_n = \frac{e}{2m_n c}(g_\ell \vec{L} + g_S \vec{S}) \]
where \( m_n \) is its mass. For a proton, \( g_\ell = 1 \) and \( g_S = 5.587 \), and for a neutron, \( g_\ell = 0 \) and \( g_S = -3.826 \).
Let \( \vec{J} = \vec{L} + \vec{S} \) be the total angular momentum. Denote by \( \ell \) and \( j \) the quantum numbers corresponding to the operators \( L^2 \) and \( J^2 \).

(a) For a given \( \ell \), what are the possible values of \( j \)?

(b) For given \( j \) and \( \ell \), show that the magnetic moment can be written as
\[ \vec{\mu}_n = \frac{e}{2m_n c} g_J \vec{J} \]
and find \( g_J \).

Problem 2
A rotator performs a hindered rotation described by the Hamiltonian
\[ H = a \vec{L}^2 + b \hbar^2 \cos(2\phi) \]
with \( a \gg b \).
Calculate the \( S \) and \( P \) energy levels of this system (with \( \ell = 0, 1 \), respectively) using first-order perturbation theory, and find the corresponding eigenstates.

You may use
\[ Y_0^0 = \frac{1}{\sqrt{4\pi}}, \quad Y_1^0 = \sqrt{\frac{3}{4\pi}} \cos \theta, \quad Y_1^{\pm 1} = \mp \sqrt{\frac{3}{8\pi}} e^{\pm i\phi} \sin \theta \]

Problem 3
A hydrogen atom is subjected to a constant electric field \( \vec{E}_0 \) that lasts for a time \( 0 < t < \tau \). If at \( t = 0 \) the atom is in the \( 2S \) state, use perturbation theory to find expressions for the state of the system \( |\psi(t)\rangle \) for \( 0 < t < \tau \) and \( t > \tau \).
What is the probability that the system will be in the \( 2P \) state for \( t > \tau \)?
You may use the spherical harmonics given above, and
\[ R_{20} = \frac{1}{\sqrt{2a_0^3}} \left( 1 - \frac{r}{2a_0} \right) e^{-r/(2a_0)}, \quad R_{21} = \frac{1}{\sqrt{24a_0^3}} \frac{r}{a_0} e^{-r/(2a_0)} \]
Problem 4
A particle \((a)\) is in a bound state. A beam of particles \((b)\) of mass \(m\), momentum \(\hbar \vec{k}_i\), and energy
\[
E_i = \frac{\hbar^2 \vec{k}_i^2}{2m}
\]
is incident on particle \((a)\).
Particle \((a)\) is described by the wavefunction \(\phi(\vec{r}_a)\) whereas each particle \((b)\) is described by the wavefunction
\[
\frac{1}{(2\pi)^{3/2}} e^{i \vec{k}_i \cdot \vec{r}_b}
\]
Particles \((a)\) and \((b)\) interact with potential energy that depends on their relative position only, \(W(\vec{r}_b - \vec{r}_a)\). Suppose that \((b)\) is scattered elastically with a final momentum \(\hbar \vec{k}_f\) whereas \((a)\) remains in the bound state \(\phi(\vec{r}_a)\).

(i) Calculate the matrix element
\[
W_{fi} = \langle \phi, a; \vec{k}_f, b | W(\vec{r}_b - \vec{r}_a) | \phi, a; \vec{k}_i, b \rangle
\]
and express it in terms of the Fourier transform \(\tilde{W}\) defined by
\[
W(\vec{r}) = \frac{1}{(2\pi)^{3/2}} \int d^3k e^{i \vec{k} \cdot \vec{r}} \tilde{W}(\vec{k})
\]
You may use
\[
\int d^3r e^{i \vec{k} \cdot \vec{r}} = (2\pi)^3 \delta^3(\vec{k})
\]

(ii) Use Fermi’s golden rule to show that the cross section is given by
\[
\frac{d\sigma}{d\Omega} = \frac{d\sigma_B}{d\Omega} F(\phi; \vec{q})
\]
where \(\frac{d\sigma_B}{d\Omega}\) is the Born cross section and \(\vec{q} = \hbar (\vec{k}_f - \vec{k}_i)\) is the momentum transfer, and find the form factor \(F\).

Problem 5
Two identical spin 1/2 particles are constrained to move in the \(x\)-direction between two infinite walls which are a distance \(L\) apart. They interact with each other through the potential
\[
W = a \vec{S}_1 \cdot \vec{S}_2 \ , \quad a < 0 \ ,
\]
where \(\vec{S}_1\) and \(\vec{S}_2\) are the two spin operators.
Find the energies of the two lowest energy levels, their degeneracies and corresponding eigenstates.