PHYSICS 522 - SPRING 2011
Final Exam

Problem 1

The magnetic moment of a nucleon is

$$\vec{\mu}_n = \frac{e}{2m_n c}(g_\ell \vec{L} + g_S \vec{S})$$

where m_n is its mass. For a proton, $g_\ell = 1$ and $g_S = 5.587$, and for a neutron, $g_\ell = 0$ and $g_S = -3.826$.
Let $\vec{J} = \vec{L} + \vec{S}$ be the total angular momentum. Denote by ℓ and j the quantum numbers corresponding to the operators L^2 and J^2.

(a) For a given ℓ, what are the possible values of j?

(b) For given j and ℓ, show that the magnetic moment can be written as

$$\vec{\mu}_n = \frac{e}{2m_n c}\, g_J \vec{J}$$

and find g_J.

Problem 2

A rotator performs a *hindered rotation* described by the Hamiltonian

$$H = a\vec{L}^2 + b\hbar^2 \cos(2\phi)$$

with $a \gg b$.
Calculate the S and P energy levels of this system (with $\ell = 0, 1$, respectively) using first-order perturbation theory, and find the corresponding eigenstates.
You may use

$$Y_0^0 = \frac{1}{\sqrt{4\pi}}\ , \quad Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta\ , \quad Y_1^{\pm 1} = \mp\sqrt{\frac{3}{8\pi}} e^{\pm i\phi}\sin\theta$$

Problem 3

A hydrogen atom is subjected to a constant electric field $\vec{\mathcal{E}}_0$ that lasts for a time $0 < t < \tau$. If at $t = 0$ the atom is in the $2S$ state, use perturbation theory to find expressions for the state of the system $|\psi(t)\rangle$ for $0 < t < \tau$ and $t > \tau$.
What is the probability that the system will be in the $2P$ state for $t > \tau$?
You may use the spherical harmonics given above, and

$$R_{20} = \frac{1}{\sqrt{2a_0^3}}\left(1 - \frac{r}{2a_0}\right)e^{-r/(2a_0)}\ , \quad R_{21} = \frac{1}{\sqrt{24a_0^3}}\frac{r}{a_0}e^{-r/(2a_0)}$$

Problem 4

A particle (a) is in a bound state. A beam of particles (b) of mass m, momentum $\hbar\vec{k}_i$, and energy

$$E_i = \frac{\hbar^2 \vec{k}_i^2}{2m}$$

is incident on particle (a).

Particle (a) is described by the wavefunction $\phi(\vec{r}_a)$ whereas each particle (b) is described by the wavefunction

$$\frac{1}{(2\pi)^{3/2}} e^{i\vec{k}_i \cdot \vec{r}_b}$$

Particles (a) and (b) interact with potential energy that depends on their relative position only, $W(\vec{r}_b - \vec{r}_a)$. Suppose that (b) is scattered elastically with a final momentum $\hbar\vec{k}_f$ whereas (a) remains in the bound state $\phi(\vec{r}_a)$.

(i) Calculate the matrix element

$$W_{fi} = \langle \phi, a; \vec{k}_f, b | W(\vec{r}_b - \vec{r}_a) | \phi, a; \vec{k}_i, b \rangle$$

and express it in terms of the Fourier transform $\widetilde{W}$ defined by

$$W(\vec{r}) = \frac{1}{(2\pi)^{3/2}} \int d^3k \, e^{i\vec{k}\cdot\vec{r}} \widetilde{W}(\vec{k})$$

You may use

$$\int d^3 r e^{i\vec{k}\cdot\vec{r}} = (2\pi)^3 \delta^3(\vec{k})$$

(ii) Use Fermi's golden rule to show that the cross section is given by

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_B}{d\Omega} \mathcal{F}(\phi; \vec{q})$$

where $\frac{d\sigma_B}{d\Omega}$ is the Born cross section and $\vec{q} = \hbar(\vec{k}_f - \vec{k}_i)$ is the momentum transfer, and find the form factor $\mathcal{F}$.

Problem 5

Two identical spin 1/2 particles are constrained to move in the x-direction between two infinite walls which are a distance L apart. They interact with each other through the potential

$$W = a\vec{S}_1 \cdot \vec{S}_2 \ , \quad a < 0 \ ,$$

where $\vec{S}_1$ and $\vec{S}_2$ are the two spin operators.

Find the energies of the <u>two</u> lowest energy levels, their degeneracies and corresponding eigenstates.