# PHYSICS 521 - FALL 2010
## Homework Set 7

*due date: Thu., December 2, 2010*

### Problem 7.1

Consider a particle in a potential $V(\rho)$ where $\vec{r} = (\rho, \phi, z)$ in cylindrical coordinates.

*(a)* Write the Hamiltonian H in cylindrical coordinates and show that it commutes with $L_z$ and $P_z$.

*(b)* Show that the eigenfunctions of the Hamiltonian can be written in the form

$$\phi_{nmk}(\vec{r}) = f_{nm}(\rho) e^{im\phi} e^{ikz}$$

Find the possible values of $m$ and $k$ and the ordinary differential equation obeyed by $f_{nm}$.

*(c)* Let $\Pi_y$ be the reflection operator with respect to the $xz$-plane,

$$\Pi_y \quad : \quad (x, y, z) \rightarrow (x, -y, z)$$

in cartesian coordinates. Describe the action of $\Pi_y$ in terms of cylindrical coordinates and show that it commutes with $H$, whereas it <u>anti</u>-commutes with $L_z$.

Furthermore, show that $\Pi_y \phi_{nmk}(\vec{r})$ is an eigenfunction of $L_z$ and find the corresponding eigenvalue.

*(d)* What can you conclude concerning the degeneracy of the energy levels of the particle from your results in part *(c)*?

Can you draw the same conclusion using only the differential equation found in part *(b)*?

### Problem 7.2

Consider an isotropic three-dimensional harmonic oscillator of mass $m$ and frequency $\omega_0$.

*(a)* Call the Hamiltonian $H_0$. Find its energy levels and their degrees of degeneracy.

Is it possible to construct a basis of eigenstates common to $H_0$, $L^2$ and $L_z$? <u>Explain.</u>

*(b)* Now assume that the particle has charge $q$ and is placed in a uniform magnetic field $\vec{B} = B\hat{z}$. Show that $\vec{B} = \vec{\nabla} \times \vec{A}$, where $\vec{A} = \frac{1}{2}\vec{B} \times \vec{r}$ and find the new Hamiltonian $H$.

Write $H$ as a sum of three commuting operators and deduce the energy levels and their degrees of degeneracy.

*(c)* If the magnetic field is *weak*, show that the effect of the diamagnetic term is negligible compared to the effect of the paramagnetic term.

Calculate the first-order correction to the first and second excited energy levels of the isotropic harmonic oscillator due to the magnetic field (Zeeman effect).

*(d)* Calculate the magnetic susceptibility $\chi$ of the ground state.