PHYSICS 521 - FALL 2010
Homework Set 6

due date: Tue., November 16, 2010

Problem 6.1

Consider a system of spin $j = 1$ which is in the state

$$|\psi\rangle = \alpha|1,1\rangle + \beta|1,0\rangle + \gamma|1,-1\rangle$$

where $|j,m\rangle$ ($m = -1, 0, +1$) are the common eigenstates of J^2 and J_z.

(a) Calculate the mean value of each of the three components of $\vec{J}$.

(b) Calculate the mean values $\langle J_x^2\rangle$, $\langle J_y^2\rangle$ and $\langle J_z^2\rangle$.

Problem 6.2

Consider a system that can have spin 0 or spin 1 (e.g., two electrons). Suppose that the system is in the state

$$|\psi\rangle = \alpha|1,1\rangle + \beta|1,0\rangle + \gamma|1,-1\rangle + \delta|0,0\rangle$$

(a) If J^2, <u>or</u> J_z, <u>or</u> J_x, <u>or</u> J_z^2 is measured, what are the possible outcomes and with what probability will each occur?

(b) What are the mean values $\langle J^2\rangle$, $\langle J_z\rangle$, $\langle J_x\rangle$ and $\langle J_z^2\rangle$?

Problem 6.3

A particle moving in three dimensions has wavefunction

$$\psi(\vec{r}) = N(x + y + z)e^{-r^2/b^2}$$

where $r^2 = x^2 + y^2 + z^2$. Let $\vec{L} = \vec{r} \times \vec{p}$ be the angular momentum.
If L^2 and L_z are measured, what are the possible outcomes and with what probability will each occur?
[Hint: Using spherical polar coordinates, express the wavefunction in terms of spherical harmonics.]

Problem 6.4

Let $\psi(\vec{r})$ be the wavefunction of a particle moving in three dimensions and $\vec{L} = \vec{r} \times \vec{p}$ the angular momentum.

(a) Can ΔL_x, ΔL_y and ΔL_z be simultaneously zero? <u>Explain.</u>

(b) Show that

$$\Delta L_x \Delta L_y \geq \frac{\hbar}{2}|\langle L_z\rangle|$$

and deduce that

$$(\Delta L_x)^2 + (\Delta L_y)^2 \geq \hbar|\langle L_z\rangle|$$

(c) Now suppose $\langle L_x \rangle = \langle L_y \rangle = 0$.

Show that the two inequalitites found in part *(b)* both become equalities if and only if $L_+|\psi\rangle = 0$, or $L_-|\psi\rangle = 0$.

Deduce that the wavefunction must be of the form

$$\psi(\vec{r}) = F(r, \sin\theta e^{\pm i\phi})$$

in spherical polar coordinates.

[Hint: Switch to coordinates $u_\pm = \sin\theta e^{\pm i\phi}$.]