PHYSICS 521 - FALL 2010
Homework Set 1

due date: Thu., September 2, 2010

Problem 1.1

To what velocity would an electron (proton) have to be slowed down, if its wavelength is to be 1 m? Are matter waves of macroscopic dimensions a real possibility? Explain.

Problem 1.2

Make an estimate of the lower bound for the distance Δx, within which an object of mass m can be localized for as long as the universe has existed ($\approx 10^{10}$ years). Compute and compare the values of this bound for an electron, a proton and the Earth.

Problem 1.3

A wavefunction $\psi(x,t)$ satisfies the Schrödinger equation with a *complex* potential,

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + [V(x) - iV_0]\psi = i\hbar\frac{\partial\psi}{\partial t}$$

where $V(x)$ is real and $V_0 > 0$.

(a) Derive the continuity equation for the probability charge (ρ) and current (j) densities.

(b) Integrate the continuity equation and show that the total probability

$$P = \int_{-\infty}^{\infty} dx\, \rho$$

decays exponentially with time.

Explain why V_0 represents *absorption*.

Problem 1.4

Show that if the potential energy $V(\vec{r})$ is changed everywhere by a constant ($V(\vec{r}) \to V(\vec{r}) + V_0$), the time-independent wavefunctions are unchanged.
What is the effect on the energy eigenvalues?

Problem 1.5

Let

$$\psi(x) = Ne^{-\lambda|x|}$$

Calculate N and the fourier transform $A(k)$.
Estimate the widths Δx of $\psi(x)$ and Δk of $A(k)$ and show

$$\Delta x \Delta k \sim 1$$

Problem 1.6

Show that, for a given energy E, the coefficients for reflection and transmission at a potential step are the same for a wave incident from the right as for a wave incident from the left.
Show also that the relative phase of the reflected to the incident amplitude is zero for reflection from a rising potential step, but π for reflection from a sharp potential drop.

Problem 1.7

A particle moves in the field of a potential

$$V(x) = V_0\theta(x) + g\delta(x) \ , \qquad \text{where} \qquad \theta(x) = \begin{cases} 0 & , \quad x \leq 0 \\ 1 & , \quad x > 0 \end{cases} \quad , \qquad V_0, g > 0$$

with energy $E > V_0$.
Write the general solution to the Schrödinger equation in the form

$$\phi(x) = \begin{cases} Ae^{ikx} + Be^{-ikx} & , \quad x < 0 \\ A'e^{ik'x} + B'e^{-ik'x} & , \quad x > 0 \end{cases}$$

and define the 2×2 *scattering matrix* $S(k, k')$ by

$$\begin{pmatrix} B \\ A' \end{pmatrix} = S(k, k') \begin{pmatrix} A \\ B' \end{pmatrix}$$

(a) Derive the elements of the matrix $S(k, k')$ and show that

$$S(-k, -k')S(k, k') = \mathbb{I}$$

(b) Calculate the transmission coefficients for a particle incident from the right and for a particle incident from the left (which have the same energy E but different speeds).