PHYSICS 521 - FALL 2007

Homework Set 3

due date: Tue., October 9, 2007

Problem 3.1

For a system described by the Hamiltonian H with eigenvalues E_i and corresponding eigenfunctions $|i\rangle$,

$$H|i\rangle = E_i|i\rangle$$

recall that the propagator is defined by

$$K(x', t'; x, t) = \sum_i e^{-iE_i(t'-t)/\hbar} \phi_i^*(x')\phi_i(x)$$

where $\phi_i(x) = \langle x|i\rangle$.
If the system is placed in a thermal bath at temperature T, it is described by the density operator

$$\rho = \frac{1}{Z}\, e^{-H/kT}$$

where $Z = \mathrm{tr} e^{-H/kT}$ is the partition function.

(a) Show that

$$Z = \int dx K(x, -i\hbar/kT; x, 0)$$

Using Feynman's expression for the propagator, describe the partition function as a sum over paths (what kind of paths?).

(b) Show that the average energy of the system is given by

$$\langle H\rangle = kT^2 \frac{\partial \ln Z}{\partial T}$$

Show that in the low temperature limit ($T \to 0$), the average energy reduces to the ground-state energy and the system is then in a pure state (the ground state).

Problem 3.2

The spin operator $\vec{S}$ acting on a spin-1/2 particle has components

$$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad , \quad S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad , \quad S_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Calculate the uncertainties ΔS_y and ΔS_z for each of the eigenstates of $\hat{n} \cdot \vec{S}$ where

$$\hat{n} = \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix}$$

and verify that they obey the Heisenberg uncertainty principle.

Problem 3.3

Do **EXERCISE 1.** (p. 341) in the textbook.

Problem 3.4

Do **EXERCISE 5.** (p. 342) in the textbook.

Problem 3.5

Do **EXERCISE 10.** (p. 344) in the textbook.

Problem 3.6

Do **EXERCISE 12.** (p. 345) in the textbook.

Problem 3.7

Do **EXERCISE 14.** (p. 347) in the textbook.