PHYSICS 232 - Solution Key to Test 3

1a. NO. You age slower if you are traveling at speed u. If $\Delta t_o = 70$ years, then for a stationary observer on Earth the time that elapses is $\Delta t = \gamma \Delta t_o$. As u approaches the speed of light, Δt goes to infinity and there is no limit in the distance ($u\Delta t$), as measured by an Earth observer, you can travel.

1b. Wien's displacement law states $\lambda T = \text{const}$. As T increases, λ decreases. In the visible spectrum, this means red $\rightarrow$ yellow $\rightarrow$ blue.

The radiation intensity is $I = \sigma T^4$, so it increases with temperature.

1c. The de Broglie wavelength is $\lambda = \frac{h}{mv}$. As the stone falls, v increases, so λ decreases.

2a. The elapsed time measured by an observer on the spaceship is $\Delta t_o = 50$ min. On the Earth, this corresponds to time

$$\Delta t = \frac{\Delta t_o}{\sqrt{1 - u^2/c^2}} = \frac{50 \text{ min}}{\sqrt{1 - 0.6^2}} = 62.5 \text{ min}$$

The distance traveled (as measured by an Earth observer) is

$$\Delta x = u\Delta t = 0.6 \times 3 \times 10^8 \times 62.5 \times 60 = 6.75 \times 10^{11} \text{ m}$$

2b. Since $\Delta t = 62.5$ min. after midnight, the spaceship passed the navigational station at Earth time 1:02:30 a.m.

2c. The signal travels at the speed of light a distance Δx, so it takes time

$$\Delta t_{\text{signal}} = \frac{\Delta x}{c} = \frac{6.75 \times 10^{11}}{3 \times 10^8} = 2250 \text{ sec} = 37.5 \text{ min}$$

The signal is detected at Earth time 1:40 a.m.

3a. From $eV_0 = hf - \phi$ and $\lambda f = c$, we have

$$\phi = \frac{hc}{\lambda} - eV_0 = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{437 \times 10^{-9}} - 1.6 \times 10^{-19} \times 1.67 = 1.86 \times 10^{-19}\ J = 1.16\ eV$$

3b. From K.E. $= eV_0$, and K.E. $= \frac{1}{2}mv^2$, we have

$$v = \sqrt{\frac{2eV_0}{m}} = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 1.67}{9.1 \times 10^{-31}}} = 7.66 \times 10^5 \text{ m/s}$$

Notice that $v \ll c$, which justifies using the non-relativistic formula for K.E.

4a. From Wien displacement law,

$$\lambda = \frac{2.9 \times 10^{-3}}{T} = \frac{2.9 \times 10^{-3}}{2800} = 1.04 \times 10^{-6}\ m$$

4b. Intensity is

$$I = \sigma T^4 = 5.67 \times 10^{-8} \times 2800^4 = 3.49 \times 10^6 \; W/m^2$$

If $A = 2$ mm^2 is the area, the total power is

$$P = IA = 3.49 \times 10^6 \times 2 \times 10^{-6} = 6.97 \; W$$

5. The energy levels are

$$E_n = \frac{n^2 h^2}{8mL^2}$$

The energy of the emitted photon is

$$E = E_4 - E_2 = \frac{16h^2}{8mL^2} - \frac{4h^2}{8mL^2} = \frac{12h^2}{8mL^2} = \frac{12 \times (6.6 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (0.7 \times 10^{-9})^2} = 1.47 \times 10^{-18} \; J$$

From $E = \frac{hc}{\lambda}$, we have

$$\lambda = \frac{hc}{E} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.47 \times 10^{-18}} = 1.35 \times 10^{-7} \; m = 135 \; nm$$