PHYSICS 232 - Solution Key to Test 2

1a. If I know $\vec{E}$ and $\vec{B}$, I can find the Poynting vector

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$

which points into the direction of propagation.

1b. From

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{s-f}{fs}$$

for a real image ($s' > 0$), we need $s > f$. This is confirmed by a ray diagram.

A convex mirror has $f < 0$, so $s' < 0$, always. So image is never real. You may confirm this with a ray diagram.

1c. Intensity is proportional to $\vec{E}^2$. Two sources creating $\vec{E}_1$, $\vec{E}_2$, respectively, have intensities $I_1 \propto \vec{E}_1^2$, $I_2 \propto \vec{E}_2^2$. Adding intensities (wrong!), we get

$$I_1 + I_2 \propto \vec{E}_1^2 + \vec{E}_2^2$$

Adding amplitudes (correct!) we get

$$(\vec{E}_1 + \vec{E}_2)^2 = \vec{E}_1^2 + \vec{E}_2^2 + 2\vec{E}_1\cdot\vec{E}_2$$

which gives $I_1 + I_2$ *plus* an additional term.

2a. Since it's an absorbing surface,

$$p_{rad} = \frac{I}{c} = \frac{1}{2}\epsilon_0 E_{max}^2 = \frac{1}{2}\times 8.85\times 10^{-12}\times 25^2 = 2.766\times 10^{-9} Pa$$

2b. $I = cp_{rad} = 2.766\times 10^{-9}\times 3\times 10^8 = 0.83$ W/m^2.

The power is

$$P = IA = 0.83\times 8\times 10^{-4} = 6.6\times 10^{-4}\ W$$

2c. The total power is

$$P_{total} = I\cdot(4\pi r^2) = 0.83\times 4\pi\times 5^2 = 261\ W$$

3a. From Snell's Law, $\sin 65^o = n\sin 43^o$, so $n = 1.33$.

3b. $\sin\theta_{crit} = \frac{1}{n}$, so $\theta_{crit} = 48.8^o$.

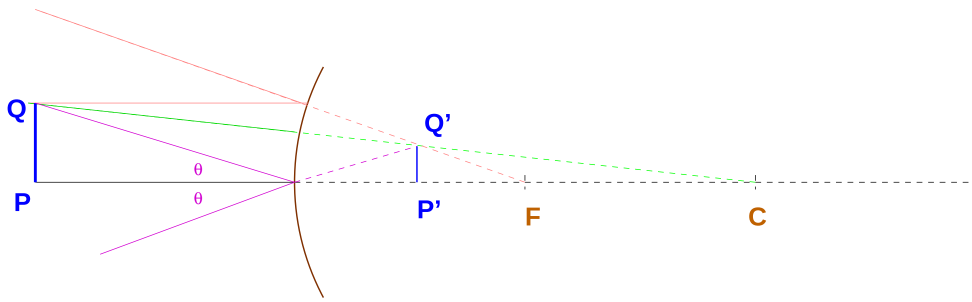
4a. A convex mirror has a negative radius of curvature, so $R = -90$ cm. We have

$$\frac{1}{s'} = \frac{2}{R} - \frac{1}{s} \quad\Rightarrow\quad s' = \frac{Rs}{2s-R} = \frac{-90\times 40}{2\times 40 - (-90)} = -21.18\ cm$$

The image is virtual.

4b. The magnification is $m = -s'/s = 21.18/40 = 0.53$.
The height of the image is $0.53 \times 3.5 = 1.85$ mm.

4c. Here is a diagram with three rays:



5. Minima are at $a \sin\theta = m\lambda$. The first minimum is at $m = 1$. We obtain

$$\theta \approx \sin\theta = \frac{\lambda}{a} = \frac{500 \times 10^{-9}}{10^{-3}} = 5 \times 10^{-4}$$

If x is its distance on the screen from the central maximum, we have $\tan\theta = x/L$ and $\theta \approx \tan\theta$, so $x = L\theta = 4 \times 5 \times 10^{-4} = 2 \times 10^{-3}$ m.

The distance between the two mimima on either side of the central maximum is double this, $2x = 2 \times 2 \times 10^{-3} = 4$ mm.