NAME:

STUDENT ID $#$:

USEFUL FORMULAS

Electromagnetic waves

$$
E = cB \quad , \quad c = \frac{1}{\sqrt{\epsilon_o \mu_o}}
$$

 $\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$, $c = 3 \times 10^8$ m/s. Poynting vector (power per unit area):

$$
\vec{S} = \frac{1}{\mu_o} \vec{E} \times \vec{B}
$$

Intensity: $I = \langle S \rangle = \frac{1}{2}$ $\frac{1}{2} \epsilon_o c E_{max}^2$. Rate of transfer of momentum per area:

$$
\frac{1}{A}\frac{dp}{dt} = \frac{S}{c}
$$

Radiation pressure:

$$
p_{rad} = I/c
$$
 (absorbing surface)

$$
p_{rad} = 2I/c
$$
 (reflecting surface)

Light

Speed of light in material: $v = \frac{c}{n}$ $\frac{c}{n}$. Snell's Law (refraction):

$$
n_a \sin \theta_a = n_b \sin \theta_b
$$

Total internal reflection for $\theta > \theta_{crit}$:

$$
\sin \theta_{crit} = \frac{n_a}{n_b}
$$

Malus's Law (intensity through polarizer):

$$
I = I_{max} \cos^2 \phi,
$$

Complete polarization ⊥ to plane of incidence:

$$
\tan \theta_p = \frac{n_b}{n_a}
$$

Geometric optics

Spherical mirror:

$$
\frac{1}{s} + \frac{1}{s'} = \frac{2}{R} = \frac{1}{f}
$$

Lateral magnification: $m = -\frac{s'}{s}$ $\frac{s'}{s}$. Spherical mirror refracting surface:

$$
\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}
$$

Lateral magnification: $m = -\frac{n_a s'}{n_a s}$ $\frac{n_{a}s^{\prime}}{n_{b}s}.$ Lens:

$$
\frac{1}{s} + \frac{1}{s'} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f}
$$

Interference and diffraction

Thin film: $2t = m\lambda$. Two sources:

$$
I = I_0 \cos^2(\pi d \sin \theta / \lambda)
$$

Maxima $(I = I_0)$ at

$$
d\sin\theta = m\lambda
$$
 $(m = ..., -2, -1, 0, 1, 2, ...)$

N-source interference:

$$
I = I_0 \frac{\sin^2(N \pi d \sin \theta / \lambda)}{\sin^2(\pi d \sin \theta / \lambda)}
$$

Principal maxima $(I = N^2 I_0)$ at

$$
d\sin\theta = m\lambda \quad (m = \dots, -2, -1, 0, 1, 2, \dots)
$$

Intensity from single aperture of width a:

$$
I = I_0 \frac{\sin^2(\pi a \sin \theta / \lambda)}{(\pi a \sin \theta / \lambda)^2}
$$

Minima at

$$
a\sin\theta = m\lambda
$$
 $(m = \dots, -2, -1, 1, 2, \dots)$

Bragg condition: $2d \sin \theta = m\lambda$. Rayleigh's criterion:

$$
\sin \theta = 1.22 \, \frac{\lambda}{D}
$$

There are five (5) problems in this test. Make sure you answer all questions in each problem for full credit. Show as much of your work as possible to receive partial credit, in case you don't come up with the right answer. Good luck!

Problem 1

- (a) By measuring the electric and magnetic fields at a point in space where there is an electromagnetic wave, can you determine the direction from which the wave came? Explain.
- (b) For what range of object positions does a concave spherical mirror form a real image? What about a convex spherical mirror?
- (c) In the calculation of intensity in an interference pattern, could you add the intensities of the waves instead of their amplitudes? Explain.

Problem 2

A totally absorbing surface of area $A = 8$ cm² faces a small source of sinusoidal electromagnetic radiation that is 5 m away. At the surface A, the electric field amplitude of the radiation is $25 \text{ V/m}.$

- (a) What is the radiation pressure exerted on the surface?
- (b) What is the power that reaches the surface A ?
- (c) What is the total power output of the source, if it is assumed to radiate uniformly in all directions?

Problem 3

A ray of light traveling in air makes a 65° angle with respect to the normal of the surface of a liquid. It travels in the liquid at a 43° angle with respect to the normal.

- (a) What is the index of refraction of the liquid?
- (b) What is the critical angle for total internal reflection?

Problem 4

An object 3.5 mm tall is placed 40 cm from the vertex of a convex spherical mirror. The radius of curvature of the mirror has magnitude 90 cm.

- (a) How far is the image from the vertex of the mirror?
- (b) What is the height of the image?
- (c) Draw a diagram showing the mirror, the object and its image including at least two rays.

Problem 5

A slit 1 mm wide is illuminated by light of wavelength 500 nm. A diffraction pattern is seen on a screen 4 m from the slit. What is the distance on the screen between the first two diffraction minima on either side of the central maximum?