PHYSICS 232 - Solution Key to Test 1

1a. $T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$. At the mountaintop, g decreases, so T increases.
The clock loses time.

1b. The fundamental is $f_1 = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$.
$f_1 \rightarrow 2f_1$ if $F \rightarrow 4F$.
$f_1 \rightarrow 4f_1$ if $F \rightarrow 16F$.

1c. $I = \frac{p_{max}^2}{2\rho v}$. If $p_{max} \rightarrow \frac{1}{2}p_{max}$, then $I \rightarrow \frac{1}{4}I$.

2a. From the equation, $A = 0.65$ m, $\omega = 5.2$ rad/s.
We deduce $f = \frac{\omega}{2\pi} = 0.83$ Hz and $T = \frac{1}{f} = 1.21$ s.

2b. Differentiating, we obtain $v = -A\omega\sin(\omega t)$, $a = -A\omega^2\cos(\omega t)$.
So $v_{max} = A\omega = 0.65 \times 5.2 = 3.38$ m/s
and $a_{max} = A\omega^2 = 0.65 \times (5.2)^2 = 17.576$ m/s^2.

2c. P.E. $= \frac{1}{2}m\omega^2 x^2 = \frac{1}{2} \times 0.85 \times (5.2)^2 \times (0.45)^2 = 2.33$ J.
Total energy $E = \frac{1}{2}m\omega^2 A^2 = \frac{1}{2} \times 0.85 \times (5.2)^2 \times (0.65)^2 = 4.86$ J.
So K.E. $= E-$ P.E. $= 4.86 - 2.33 = 2.53$ J.

3a. Speed $v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{1.6\times10^9}{918}} = 1.32 \times 10^3$ m/s.
From $v = L/t$, we deduce $t = \frac{L}{v} = \frac{15\times10^{-2}}{1.32\times10^3} = 1.14 \times 10^{-4}$ s.

3b. From $v = \lambda f$, we deduce $\lambda = \frac{v}{f} = \frac{1.32\times10^3}{8.5} = 155.3$ m.

3c. Pressure amplitude

$$p_{max} = BkA = B\frac{2\pi}{\lambda}A = 1.6 \times 10^9 \times \frac{2\pi}{155.3} \times 2.5 \times 10^{-3} = 1.6 \times 10^5 \text{ Pa}$$

Intensity $I = \frac{p_{max}^2}{2\rho v} = \frac{(1.6\times10^5)^2}{2\times918\times1.32\times10^3} = 1.1 \times 10^4$ W/m^2.

4a. By definition, $I = \frac{\text{Power}}{\text{Area}}$, so Power $= I\times$ Area.
From $\beta = (10\ dB)\log\frac{I}{10^{-12}}$, we deduce

$$I = 10^{-12} \times 10^{6.5} = 3.16 \times 10^{-6}\ W/m^2$$

So Power $= 3.16 \times 10^{-6} \times (2.5 \times 0.75) = 5.9 \times 10^{-6}$ W.

4b. We have $I \propto 1/r^2$, so

$$\frac{I_{\text{midpoint}}}{I} = \frac{r^2}{(r/2)^2} = 4$$

so $I_{\text{midpoint}} = 4I = 4 \times 3.16 \times 10^{-6} = 1.264 \times 10^{-5}$ W/m^2.

4c. From $I = \frac{\text{Total Power}}{4\pi r^2}$, we deduce

$$\text{Total Power} = \text{I} \times (4\pi\text{r}^2) = 3.16 \times 10^{-6} \times 4\pi \times (25)^2 = 0.025 \text{ W}$$