PHYSICS 232 - Solution Key to Sample Test 3

1. $v = 0.9c$, $u = 0.7c$ and

$$v' = \frac{v-u}{1-uv/c^2} = \frac{0.9-0.7}{1-0.9\times 0.7}\,c = 0.54c$$

2a.

$$\begin{aligned} E &= \sqrt{m^2c^4 + p^2c^2} \\ &= \sqrt{(3.32\times 10^{-27})^2 \times (3\times 10^8)^4 + (8.25\times 10^{-19}\times 3\times 10^8)^2} \\ &= 3.88\times 10^{10}\ J \end{aligned}$$

2b. K.E. $= E - mc^2 = 3.88\times 10^{10} - 3.32\times 10^{-27}\times (3\times 10^8)^2 = 8.9\times 10^{-11}$ J.

2c.

$$\frac{\text{K.E.}}{mc^2} = \frac{8.9\times 10^{-11}}{3.32\times 10^{-27}\times (3\times 10^8)^2} = 0.3$$

3. $L = pr = n\frac{h}{2\pi}$, so $p = n\frac{h}{2\pi r}$. Wavelength: $\lambda = \frac{h}{p} = \frac{2\pi r}{n}$ and $r = n^2 a_0$, so $\lambda = 2\pi n a_0$.
For $n = 1$, $\lambda = 2\pi a_0 = 2\pi \times 5.29\times 10^{-11} = 3.32\times 10^{-10}$m, $\frac{\lambda}{2\pi r} = 1$.
For $n = 4$, $\lambda = 8\pi a_0 = 8\pi \times 5.29\times 10^{-11} = 1.33\times 10^{-9}$m, $\frac{\lambda}{2\pi r} = 1/4$.

4a. K.E.$_{max} = eV_0 = 1.25$ eV.

4b. K.E.$_{max} = \frac{1}{2}mv^2$, so

$$v = \sqrt{\frac{2\text{K.E.}_{max}}{m}} = \sqrt{\frac{2\times 1.25\times 1.6\times 10^{-19}}{9.1\times 10^{-31}}} = 6.6\times 10^5\ m/s$$

5a. The electron is in a bound state. We need to supply (positive) energy to free the electron (and ionize the hydrogen atom). If we supply just enough energy to free it, it will have zero velocity and therefore zero energy. So it had to have negative energy to begin with.

5b. When electrons go through an apperture, an inteference pattern forms. We can calculate the spread in the transverse momentum Δp_y from the interference pattern. If a is the width of the aperture, Heisenberg's uncertainty principle states

$$a\Delta p_y \geq \frac{h}{2\pi}$$

We may vary the width of the aperture and see if the product $a\Delta p_y$ stays above the stated value.

5c. Shine X-rays on a target. After X-rays go through the target, we observe X-rays of the original wavelength, as well as of a slightly shifted wavelength. This can be understood (and the shifted wavelength can be calculated) as a collision between an X-ray photon and an electron.

It is not observable with visible light, because the shift in the wavelength, about

$$\frac{h}{mc} = \frac{6.6\times 10^{-34}}{9.1\times 10^{-31}\times 3\times 10^8} = 2.4\times 10^{-12} m$$

is too small to be observable in that case ($\lambda \sim 500$ nm).