PHYSICS 232 - Solution Key to the Final Exam

1a. The thicker strings have larger linear density μ. From

$$f_1 = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$$

we deduce that thicker strings produce lower frequency.

1b. We have

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad \Rightarrow \quad s' = \frac{sf}{s-f}$$

- For a concave mirror $f > 0$.
  - For $s > f$, $s' > 0$ (real image) and magnification $m = -s'/s < 0$ (inverted).
  - For $s < f$, $s' < 0$ (virtual image) and $m > 0$ (erect).
- On the convex side, $f < 0$, so $s' < 0$ (virtual image) always and $m > 0$ (erect).

The above may also be deduced from ray diagrams - equally acceptable.

1c. The first minimum in the intensity of a wave through an aperture of width a is at

$$\sin\theta = \frac{\lambda}{a} = \frac{v}{af}$$

If f is high, we need a smaller a (tweeter) to maintain the same θ as for low f.

2a. The area between x_1 and x_2,

$$\int_{x_1}^{x_2} |\psi(x)|^2 dx$$

is the probability to find the particle between x_1 and x_2.

The area between 0 and L is the total probability, so

$$\int_0^L |\psi(x)|^2 dx = 1$$

2b. The ionization energy of Hydrogen is

$$E_1 = 13.6\ eV$$

A transition between levels n and n' produces a photon of energy (assuming $n' > n$)

$$E = \left(\frac{1}{n^2} - \frac{1}{n'^2}\right) E_1$$

which is always less than E_1. The photon with the minimum wavelength that can be produced has

$$E_1 = \frac{hc}{\lambda} \quad \Rightarrow \quad \lambda = \frac{hc}{E_1} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{13.6 \times 1.6 \times 10^{-19}} = 91\ nm$$

X-rays have much smaller wavelengths, so they cannot be produced.

2c. Bonds of organic molecules have energies below 0.5 eV, so transitions emit photons of energies less than $E = 0.5$ eV. These photons have wavelengths larger than

$$\lambda = \frac{hc}{E} = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{0.5 \times 1.6 \times 10^{-19}} = 2.5\ \mu m$$

well above the optical range (400-700 nm). Most of the wavelengths lie in the radio frequency range.

3a. Use

$$f_L = \frac{v + v_L}{v + v_S}\ f_S$$

The listener is the wall, so $v_L = 0$ and we want $f_L = 650$ Hz. The source (soprano) is moving toward the wall, so $v_S = -35$ m/s and

$$f_S = \frac{v + v_S}{v + v_L}\ f_L = \frac{340 - 35}{340 + 0}\ 650 = 583\ Hz$$

3b. Now the source is the wall so $v_S = 0$ and $f_S = 650$ Hz. The listener (soprano) is moving toward the wall, so $v_L = +35$ m/s and

$$f_L = \frac{v + v_L}{v + v_S}\ f_S = \frac{340 + 35}{340 + 0}\ 650 = 717\ Hz$$

4a. Use Rayleigh's criterion,

$$\sin\theta = 1.22\ \frac{\lambda}{D}$$

For Hubble: use $\lambda = 400$ nm (smallest wavelength). θ is very small, so

$$\theta \approx \sin\theta = 1.22\ \frac{400 \times 10^{-9}}{2.4} = 2.0 \times 10^{-7}\ \text{rad}$$

The smallest crater that can be seen has size

$$y = L\theta = 383{,}000{,}000 \times 2.0 \times 10^{-7} = 77\ m$$

For Arecibo:

$$\theta = 1.22\ \frac{75 \times 10^{-2}}{305} = 2.5 \times 10^{-3}\ \text{rad}$$

$$y = 383{,}000{,}000 \times 2.5 \times 10^{-3} = 940\ km$$

4b. In this case, $y = 40$ cm and using θ found above,

$$L = \frac{y}{\theta} = \frac{40 \times 10^{-2}}{2.0 \times 10^{-7}} = 2{,}000\ km$$

5. Let $L = 56$ light years and $u = 0.99c$. The time it takes the probe to reach the star, as measured by an Earth observer, is

$$\Delta t = \frac{L}{u}$$

Laika ages according to her proper time,

$$\Delta t_0 = \frac{\Delta t}{\gamma} = \frac{L}{u}\sqrt{1 - \frac{u^2}{c^2}} = \frac{(56\ years) \times c}{0.99c}\sqrt{1 - 0.99^2} = 8\ years$$

So Laika will only be $2 + 8 = 10$ years old when the probe reaches the star.

6a. $n = 4$, so

$$E = -\frac{13.6\ eV}{n^2} = -\frac{13.6\ eV}{4^2} = -0.85\ eV$$

6b. From

$$U = m_\ell\ \frac{eh}{4\pi m}\ B$$

the splitting is

$$\Delta U = \frac{eh}{4\pi m}\ B$$

Solving for the magnetic field with $\Delta U = 3.5 \times 10^{-5}$ eV,

$$B = \frac{4\pi m \Delta U}{eh} = \frac{4\pi \times 9.1 \times 10^{-31} \times 3.5 \times 10^{-5} \times 1.6 \times 10^{-19}}{1.6 \times 10^{-19} \times 6.6 \times 10^{-34}} = 0.6\ T$$

6c. $\ell = 1$, so $m_\ell = -1, 0, 1$, so 3 levels.

7. Let $r_1 = -\frac{dN}{dt} = 9500$ decays/min at $t = 0$ and $r_2 = -\frac{dN}{dt} = 8800$ decays/min at $t =$ 30 days. We have

$$\frac{dN}{dt} = -\lambda N$$

so

$$r_1 = -\lambda N_0 \quad , \qquad r_2 = -\lambda N(t) = -\lambda N_0 e^{-\lambda t} = r_1 e^{-\lambda t}$$

Solving for λ,

$$\lambda = \frac{1}{t} \ln \frac{r_1}{r_2} = \frac{1}{30 \times 24 \times 3,600} \ln \frac{9500}{8800} = 2.95 \times 10^{-8}\ s^{-1}$$

The half-life is

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{2.95 \times 10^{-8}} = 2.35 \times 10^7\ s = 272\ days$$