PHYSICS 232 – FINAL EXAM

NAME:

STUDENT ID #:

USEFUL CONSTANTS

$$\epsilon_0 = 8.85 \times 10^{-12} C^2/(N \cdot m^2)$$

$$c = 3 \times 10^8 m/s$$

$$h = 6.6 \times 10^{-34} J \cdot s$$

$$k = 1.38 \times 10^{-23} J/K$$

$$e = 1.6 \times 10^{-19} C$$

$$m_e = 9.1 \times 10^{-31} kg$$

USEFUL FORMULAS

Periodic motion

$$f = \frac{1}{T} \quad , \quad \omega = 2\pi f$$

Spring: $F = -kx$, $\omega = \sqrt{k/m}$
Conservation of Energy:

$$E = \frac{1}{2}mv^2 + \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}m\omega^2 A^2$$

Simple Pendulum: $\omega = \sqrt{g/L}$
Damping force $F = -bv$, damped oscillation

$$x = Ae^{-bt/(2m)} \cos\sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}\, t$$

Driving force of frequency ω_0, resonance:

$$A = \frac{F_{max}}{\sqrt{(k - m\omega_0^2)^2 + b^2\omega_0^2}}$$

Mechanical waves

$$v = \lambda f = \frac{\omega}{k} = \sqrt{\frac{F}{\mu}} \quad , \quad k = \frac{2\pi}{\lambda}$$

Average power: $P_{av} = \frac{1}{2}\sqrt{\mu F}\omega^2 A^2$
Standing wave:

$$y(x,t) = A_{sw} \sin kx \ \cos \omega t$$

On a string of length L with both ends fixed,

$$f_n = n\,\frac{v}{2L} = nf_1, \quad (n = 1, 2, 3, \dots)$$

Sound

Pressure amplitude: $p_{max} = BkA$
Speed:

$$v = \sqrt{\frac{B}{\rho}} \ \text{(fluid)}, \quad \sqrt{\frac{Y}{\rho}} \ \text{(solid rod)},$$

$$v = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\frac{\gamma RT}{M}} \ \text{(ideal gas)}$$

Intensity:

$$I = \frac{p_{max}^2}{2\rho v} = \frac{\text{total power}}{4\pi r^2}$$

Sound intensity level:

$$\beta = (10dB) \log\frac{I}{I_o} \ , \qquad I_o = 10^{-12}\mathrm{W/m^2}$$

Pipe open at both ends,

$$f_n = \frac{nv}{2L} \quad (n = 1, 2, 3, \dots)$$

Pipe open at one end and closed at the other (stopped pipe),

$$f_n = \frac{nv}{4L} \quad (n = 1, 3, 5, \dots)$$

Beat frequency ($f_a > f_b$): $f_{beat} = f_a - f_b$
The Doppler effect (S: source, L: listener):

$$f_L = \frac{v + v_L}{v + v_S} f_S$$

Source moving with speed $v_S > v$: $\sin\alpha = \frac{v}{v_S}$

Electromagnetic waves

$$E = cB \quad , \quad c = \frac{1}{\sqrt{\epsilon_o \mu_o}}$$

Poynting vector (power per unit area):

$$\vec{S} = \frac{1}{\mu_o} \vec{E} \times \vec{B}$$

Intensity: $I = \langle S \rangle = \frac{1}{2} \epsilon_o c E_{max}^2$.
Rate of transfer of momentum per area:

$$\frac{1}{A} \frac{dp}{dt} = \frac{S}{c}$$

Radiation pressure:

$$p_{rad} = I/c \quad \text{(absorbing surface)}$$

$$p_{rad} = 2I/c \quad \text{(reflecting surface)}$$

Speed of light in material: $v = \frac{c}{n}$.
Snell's Law (refraction): $n_a \sin \theta_a = n_b \sin \theta_b$
Total internal reflection: $\sin \theta_{crit} = n_a / n_b$
Malus's Law (polarizer): $I = I_{max} \cos^2 \phi$
Brewster angle: $\tan \theta_p = n_b / n_a$
Spherical mirror:

$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R} = \frac{1}{f}$$

Lateral magnification: $m = -\frac{s'}{s}$.
Spherical mirror refracting surface:

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$

Lateral magnification: $m = -\frac{n_a s'}{n_b s}$.
Lens:

$$\frac{1}{s} + \frac{1}{s'} = (n-1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right) = \frac{1}{f}$$

Interference and diffraction

Thin film: $2t = m\lambda$.
Two sources:

$$I = I_0 \cos^2(\pi d \sin \theta / \lambda)$$

Maxima ($I = I_0$) at

$$d \sin \theta = m\lambda \quad (m = \ldots, -2, -1, 0, 1, 2, \ldots)$$

N-source interference:

$$I = I_0 \frac{\sin^2(N \pi d \sin \theta / \lambda)}{\sin^2(\pi d \sin \theta / \lambda)}$$

Principal maxima ($I = N^2 I_0$) at

$$d \sin \theta = m\lambda \quad (m = \ldots, -2, -1, 0, 1, 2, \ldots)$$

Intensity from single aperture of width a:

$$I = I_0 \frac{\sin^2(\pi a \sin \theta / \lambda)}{(\pi a \sin \theta / \lambda)^2}$$

Minima at

$$a \sin \theta = m\lambda \quad (m = \ldots, -2, -1, 1, 2, \ldots)$$

Bragg condition: $2d \sin \theta = m\lambda$.
Rayleigh's criterion: $\sin \theta = 1.22 \frac{\lambda}{D}$

Relativity

Time dilation:

$$\Delta t == \gamma \Delta t_o \ , \quad \gamma = \frac{1}{\sqrt{1 - u^2/c^2}}$$

Length contraction: $l = \frac{l_o}{\gamma}$
Lorentz transformation

$$x' = \gamma(x - ut) \ , \quad t' = \gamma(t - ux/c^2) \ .$$

Addition of velocities:

$$v' = \frac{v - u}{1 - uv/c^2}$$

Doppler effect:

$$f = \sqrt{\frac{c+u}{c-u}} f_o$$

Momentum and energy:

$$\vec{p} = \gamma m \vec{v} \ , \quad E = \sqrt{m^2 c^4 + p^2 c^2} = \gamma m c^2$$

Quantum Mechanics

Energy of a photon: $E = hf = \frac{hc}{\lambda}$
Photoelectric effect: $eV_0 = hf - \phi$
Bohr model of the H atom:

$$L = m_e v r = n \frac{h}{2\pi} \quad r = n^2 a_0$$

where $a_0 = \frac{\epsilon_0 h^2}{\pi m_e e^2} = 5.29 \times 10^{-11} m$,

$$E_n = -\frac{m_e e^4}{8 h^2 \epsilon_0^2 n^2} = -\frac{13.6 \ eV}{n^2}$$

X-rays from electron impact on a target:

$$eV_{AC} = hf_{max} = \frac{hc}{\lambda_{min}}$$

Compton scattering:

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos \phi)$$

Stefan-Boltzmann law:

$$I = \sigma T^4 \quad \sigma = 5.67 \times 10^{-8} W/m^2 \cdot K^4$$

Wien displacement law:

$$\lambda T = 2.9 \times 10^{-3} m \cdot K$$

Planck radiation law:

$$I(\lambda) = \frac{2\pi h c^2}{\lambda^5 (e^{hc/\lambda kT} - 1)}$$

de Broglie wavelength

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Heisenberg uncertainty principle:

$$\Delta p_x \Delta x \geq \hbar \quad , \quad \Delta E \Delta t \geq \hbar \quad , \quad \hbar = \frac{h}{2\pi}$$

Schrödinger equation:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x)$$

Infinitely deep square potential well:

$$E_n = \frac{n^2 h^2}{8mL^2} \quad , \quad \psi_n = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

Harmonic oscillator:

$$E_n = \left(n + \frac{1}{2}\right) \hbar\omega \quad (n = 0, 1, 2, \ldots)$$

Energy levels of the H atom:

$$E_n = -\frac{me^4}{8h^2 \epsilon_0^2 n^2} = -\frac{13.6\ eV}{n^2}$$

Orbital angular momentum has magnitude

$$L = \sqrt{\ell(\ell+1)}\frac{h}{2\pi} \quad (\ell = 0, 1, \ldots, n-1)$$

and z-component

$$L_z = m_\ell \frac{h}{2\pi} \quad m_\ell = -\ell, \ldots, 0, \ldots, \ell$$

Similarly for spin ($s = 1/2$, $m_s = \pm 1/2$).
In a uniform magnetic field in the z-direction, the extra energy is

$$U = m_\ell \frac{eh}{4\pi m}\ B$$

For spin, $U = 2m_s \frac{eh}{4\pi m}\ B$ (Dirac).
Binding energy in nucleus:

$$E_B = (ZM_H + Nm_n - {}^A_Z M)c^2$$

Nuclear decay:

$$N(t) = N_0 e^{-\lambda t} \quad , \quad T_{mean} = \frac{1}{\lambda} = \frac{T_{1/2}}{\ln 2}$$