PHYSICS 232 – FINAL EXAM

NAME:

STUDENT ID #:

USEFUL CONSTANTS

$$\epsilon_0 = 8.85 \times 10^{-12} C^2/(N \cdot m^2)$$

$$c = 3 \times 10^8 m/s$$

$$h = 6.6 \times 10^{-34} J \cdot s$$

$$k = 1.38 \times 10^{-23} J/K$$

$$e = 1.6 \times 10^{-19} C$$

$$m_e = 9.1 \times 10^{-31} kg$$

USEFUL FORMULAS

Periodic motion

$$f = \frac{1}{T} \quad , \quad \omega = 2\pi f$$

Spring: $F = -kx$, $\omega = \sqrt{k/m}$
Conservation of Energy:

$$E = \frac{1}{2}mv^2 + \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}m\omega^2 A^2$$

Simple Pendulum: $\omega = \sqrt{g/L}$
Damping force $F = -bv$, damped oscillation

$$x = Ae^{-bt/(2m)} \cos \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}\, t$$

Driving force of frequency ω_0, resonance:

$$A = \frac{F_{max}}{\sqrt{(k - m\omega_0^2)^2 + b^2\omega_0^2}}$$

Mechanical waves

$$v = \lambda f = \frac{\omega}{k} = \sqrt{\frac{F}{\mu}} \quad , \quad k = \frac{2\pi}{\lambda}$$

Average power: $P_{av} = \frac{1}{2}\sqrt{\mu F}\omega^2 A^2$
Standing wave:

$$y(x,t) = A_{sw} \sin kx \, \cos \omega t$$

On a string of length L with both ends fixed,

$$f_n = n\frac{v}{2L} = nf_1, \quad (n = 1, 2, 3, \dots)$$

Sound

Pressure amplitude: $p_{max} = BkA$
Speed:

$$v = \sqrt{\frac{B}{\rho}} \text{ (fluid)}, \quad \sqrt{\frac{Y}{\rho}} \text{ (solid rod)},$$

$$v = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\frac{\gamma RT}{M}} \text{ (ideal gas)}$$

Intensity:

$$I = \frac{p_{max}^2}{2\rho v} = \frac{\text{total power}}{4\pi r^2}$$

Sound intensity level:

$$\beta = (10dB)\log\frac{I}{I_o} \quad , \quad I_o = 10^{-12}\mathrm{W/m^2}$$

Pipe open at both ends,

$$f_n = \frac{nv}{2L} \quad (n = 1, 2, 3, \dots)$$

Pipe open at one end and closed at the other (stopped pipe),

$$f_n = \frac{nv}{4L} \quad (n = 1, 3, 5, \dots)$$

Beat frequency ($f_a > f_b$): $f_{beat} = f_a - f_b$
The Doppler effect (S: source, L: listener):

$$f_L = \frac{v + v_L}{v + v_S} f_S$$

Source moving with speed $v_S > v$: $\sin\alpha = \frac{v}{v_S}$

Electromagnetic waves

$$E = cB \quad , \quad c = \frac{1}{\sqrt{\epsilon_o \mu_o}}$$

Poynting vector (power per unit area):

$$\vec{S} = \frac{1}{\mu_o} \vec{E} \times \vec{B}$$

Intensity: $I = \langle S \rangle = \frac{1}{2} \epsilon_o c E_{max}^2$.

Rate of transfer of momentum per area:

$$\frac{1}{A} \frac{dp}{dt} = \frac{S}{c}$$

Radiation pressure:

$$p_{rad} = I/c \quad \text{(absorbing surface)}$$

$$p_{rad} = 2I/c \quad \text{(reflecting surface)}$$

Speed of light in material: $v = \frac{c}{n}$.

Snell's Law (refraction): $n_a \sin\theta_a = n_b \sin\theta_b$

Total internal reflection: $\sin\theta_{crit} = n_a/n_b$

Malus's Law (polarizer): $I = I_{max} \cos^2\phi$

Brewster angle: $\tan\theta_p = n_b/n_a$

Spherical mirror:

$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R} = \frac{1}{f}$$

Lateral magnification: $m = -\frac{s'}{s}$.

Spherical mirror refracting surface:

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$

Lateral magnification: $m = -\frac{n_a s'}{n_b s}$.

Lens:

$$\frac{1}{s} + \frac{1}{s'} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f}$$

Interference and diffraction

Thin film: $2t = m\lambda$.

Two sources:

$$I = I_0 \cos^2(\pi d \sin\theta/\lambda)$$

Maxima ($I = I_0$) at

$$d\sin\theta = m\lambda \quad (m = \ldots, -2, -1, 0, 1, 2, \ldots)$$

N-source interference:

$$I = I_0 \frac{\sin^2(N\pi d \sin\theta/\lambda)}{\sin^2(\pi d \sin\theta/\lambda)}$$

Principal maxima ($I = N^2 I_0$) at

$$d\sin\theta = m\lambda \quad (m = \ldots, -2, -1, 0, 1, 2, \ldots)$$

Intensity from single aperture of width a:

$$I = I_0 \frac{\sin^2(\pi a \sin\theta/\lambda)}{(\pi a \sin\theta/\lambda)^2}$$

Minima at

$$a\sin\theta = m\lambda \quad (m = \ldots, -2, -1, 1, 2, \ldots)$$

Bragg condition: $2d\sin\theta = m\lambda$.

Rayleigh's criterion: $\sin\theta = 1.22\,\frac{\lambda}{D}$

Relativity

Time dilation:

$$\Delta t = \gamma \Delta t_o \ , \quad \gamma = \frac{1}{\sqrt{1 - u^2/c^2}}$$

Length contraction: $l = \frac{l_o}{\gamma}$

Lorentz transformation

$$x' = \gamma(x - ut) \ , \quad t' = \gamma(t - ux/c^2) \ .$$

Addition of velocities:

$$v' = \frac{v - u}{1 - uv/c^2}$$

Doppler effect:

$$f = \sqrt{\frac{c+u}{c-u}} f_o$$

Momentum and energy:

$$\vec{p} = \gamma m \vec{v} \ , \quad E = \sqrt{m^2c^4 + p^2c^2} = \gamma mc^2$$

Quantum Mechanics

Energy of a photon: $E = hf = \frac{hc}{\lambda}$

Photoelectric effect: $eV_0 = hf - \phi$

Bohr model of the H atom:

$$L = m_e v r = n\frac{h}{2\pi} \quad r = n^2 a_0$$

where $a_0 = \frac{\epsilon_0 h^2}{\pi m_e e^2} = 5.29 \times 10^{-11} m$,

$$E_n = -\frac{m_e e^4}{8h^2\epsilon_0^2 n^2} = -\frac{13.6\ eV}{n^2}$$

X-rays from electron impact on a target:

$$eV_{AC} = hf_{max} = \frac{hc}{\lambda_{min}}$$

Compton scattering:

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$$

Stefan-Boltzmann law:

$$I = \sigma T^4 \quad \sigma = 5.67 \times 10^{-8} W/m^2 \cdot K^4$$

Wien displacement law:

$$\lambda T = 2.9 \times 10^{-3} m \cdot K$$

Planck radiation law:

$$I(\lambda) = \frac{2\pi h c^2}{\lambda^5 (e^{hc/\lambda k T} - 1)}$$

de Broglie wavelength

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Heisenberg uncertainty principle:

$$\Delta p_x \Delta x \geq \hbar \quad , \qquad \Delta E \Delta t \geq \hbar \quad , \qquad \hbar = \frac{h}{2\pi}$$

Schrödinger equation:

$$-\frac{\hbar^2}{2m} \frac{d^2 \psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x)$$

Infinitely deep square potential well:

$$E_n = \frac{n^2 h^2}{8 m L^2} \quad , \qquad \psi_n = \sqrt{\frac{2}{L}} \sin \frac{n \pi x}{L}$$

Harmonic oscillator:

$$E_n = \left(n + \frac{1}{2} \right) \hbar \omega \qquad (n = 0, 1, 2, \dots)$$

Energy levels of the H atom:

$$E_n = -\frac{m e^4}{8 h^2 \epsilon_0^2 n^2} = -\frac{13.6\ eV}{n^2}$$

Orbital angular momentum has magnitude

$$L = \sqrt{\ell(\ell+1)} \frac{h}{2\pi} \qquad (\ell = 0, 1, \dots, n-1)$$

and z-component

$$L_z = m_\ell \frac{h}{2\pi} \qquad m_\ell = -\ell, \dots, 0, \dots, \ell$$

Similarly for spin ($s = 1/2$, $m_s = \pm 1/2$).
In a uniform magnetic field in the z-direction, the extra energy is

$$U = m_\ell \frac{eh}{4\pi m} B$$

For spin, $U = 2 m_s \frac{eh}{4\pi m} B$ (Dirac).
Binding energy in nucleus:

$$E_B = (Z M_H + N m_n - {}^{A}_{Z}M) c^2$$

Nuclear decay:

$$N(t) = N_0 e^{-\lambda t} \quad , \qquad T_{mean} = \frac{1}{\lambda} = \frac{T_{1/2}}{\ln 2}$$

There are seven (7) problems in this exam. Make sure you answer all questions in each problem for full credit. Show as much of your work as possible to receive partial credit, in case you don't come up with the right answer. Good luck!

George Siopsis - 12/12/05

Problem 1

(a) The four strings on a violin have different thicknesses, but are all under approximately the same tension. How does the fundamental vibration frequency compare for the thick versus the thin strings? *Explain.*

(b) You look at your reflection in the concave side of a shiny spoon. Is it right side up or inverted?

Does it matter how far your face is from the spoon?

What if you look in the convex side?

(c) Low-frequency sounds are produced by the *woofer*, which is a speaker with large diameter; the *tweeter*, a speak of smaller diameter, produces high-frequency sounds. Use diffraction ideas to explain why the tweeter is more effective for distributing high-frequency sounds uniformly over a room than is the woofer.

Problem 2

(a) For a particle in a box of size L described by the wavefunction $\psi(x)$ $(0 < x < L)$, what is the physical significance of the area under the graph of $|\psi(x)|^2$ between x_1 and x_2?

What is the area under the graph of $|\psi(x)|^2$ between 0 and L? *Explain.*

(b) Can a Hydrogen atom emit x-rays? If so, how? If not, why not?

(c) Various organic molecules have been discovered in interstellar space. Why were these discoveries made with radio telescopes rather than optical telescopes?

Problem 3

A man marries a great Wagnerian soprano but alas, he discovers he cannot stand Wagnerian opera. In order to save his eardrums, the unhappy man decides he must silence his lark-like wife for good. His plan is to tie her to the front of his car and send car and soprano speeding toward a brick wall. This soprano, however, is quite shrewd, having studied physics in her student days at the Conservatory. She realizes that this wall has a resonant frequency of 650 Hz, which means that if a continuous sound wave of this frequency hits the wall, it will fall down and she will be saved to sing again. The car is heading toward the wall at a high speed of 35 m/s. The speed of sound in air is 340 m/s.

(a) At what frequency must the soprano sing so that the wall will crumble?

(b) What frequency will the soprano hear reflected from the wall just before it crumbles?

Problem 4

The Hubble Space Telescope has an aperture of 2.4 m and focuses visible light (400 nm to 700 nm). The Arecibo Radio Telescope in Puerto Rico is 305 m in diameter and focuses radio waves of wavelength 75 cm.

(a) Under optimal viewing conditions, what is the smallest crater that each of these telescopes could resolve on the Moon (at distance 383,000 km from Earth)?

(b) If the Hubble telescope were to be converted to surveillance use, what is the highest orbit above the surface of the Earth it could have and still be able to resolve a 40 cm license plate of a car on the ground? Assume optimal viewing conditions, so that resolution is diffraction limited.

Problem 5

A space probe is sent to the vicinity of a star which is 56 light years from Earth. The probe travels with a speed 0.99c. A 2-year-old dog named Laika is on board when the probe leaves the Earth. What is Laika's biological age when the probe reaches the star (assuming she survives)?

Problem 6

A Hydrogen atom in a $4p$ state is placed in a uniform external magnetic field. Consider the interaction of the magnetic field with the atom's orbital magnetic dipole moment.

(a) What is the energy of the $4p$ state before the magnetic field is applied?

(b) What magnetic field magnitude is required to split the $4p$ state into multiple levels with an energy difference of 3.5×10^{-5} eV between adjacent levels?

(c) How many levels will there be?

Problem 7

Measurements on a certain quantity of the radioactive isotope ^{57}Co tell you that the decay rate decreases from 9500 decays/min to 8800 decays/min in 30 days. What is the half-life of ^{57}Co?