PHYSICS 231 - Solution Key to Test 2

1a. Before the metal is inserted, the capacitance is $C = \epsilon_0 A/d$. After it is inserted, we have effectively two capacitors connected in series of capacitances $C_1 = \epsilon_0 A/d_1$ and $C_2 = \epsilon_0 A/d_2$, respectively. The capacitance is now

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d_1 + d_2}{\epsilon_0 A}$$

Since $d > d_1 + d_2$, we have $1/C > 1/C'$, so $C' > C$. The capacitance increases.

1b. In an open circuit, there is no current, so the voltage is $\mathcal{E}$. If the battery is short-circuited, then $\mathcal{E} = Ir$, so $r = \mathcal{E}/I$. The rule of thumb is correct.

1c. With or without R_2, the current through R_1 is

$$I_1 = \frac{\mathcal{E}}{R_1}$$

2a. The area of each plate is $A = a^2 = 2.25\ m^2$. The capacitance is

$$C = \epsilon_0 \frac{A}{d} = 1.66 \times 10^{-8}\ F$$

2b. The voltage is

$$V = \frac{Q}{C} = 84.37\ V$$

The electric field is

$$E = \frac{V}{d} = 7.03 \times 10^4 V/m$$

2c. The energy is

$$U = \frac{Q^2}{2C} = 5.9 \times 10^{-5}\ J$$

2d. If the separation is $d' = 3.6$ mm, the capacitance is

$$C' = \epsilon_0 \frac{A}{d'} = 5.53 \times 10^{-9}\ F$$

and the energy is

$$U' = \frac{Q^2}{2C'} = 1.77 \times 10^{-4}\ J$$

The work you have to do is

$$W = U' - U = 1.18 \times 10^{-4} J$$

3a. R_1 and R_2 are in series with total resistance

$$R_x = R_1 + R_2 = 6\,\Omega$$

R_x and R_3 are in parallel with total resistance

$$\frac{1}{R} = \frac{1}{R_x} + \frac{1}{R_3} \quad \Rightarrow \quad R = 4\,\Omega$$

The current is

$$I = \frac{\mathcal{E}}{r + R} = 2.4\ A$$

I splits into I_x (through R_x) and I_3 (through R_3). R, R_x and R_3 all have the same voltage, so

$$V_{ab} = IR = I_x R_x = I_3 R_3$$

so

$$I_3 = I\frac{R}{R_3} = 0.8\ A$$

and the current through R_1 and R_2 is

$$I_x = I\frac{R}{R_x} = 1.6\ A$$

3b.

$$V_{ab} = IR = \mathcal{E} - Ir = 9.6\ V$$

3c. There will be no current through R_1 and R_2 and the current though R_3 will be

$$I' = \frac{\mathcal{E}}{R_3 + r} = 0.92\ A$$