PHYSICS 231 - Solution Key to Test 1

1a. The force is $F = qE$, so the proton and electron experience *opposite* forces (eE and $-eE$, respectively).

From $F = ma$, we see that the acceleration of the proton is *smaller* than the acceleration of the electron, because $m_e < m_p$.

They move in *opposite* directions.

1b. From Gauss's Law, the flux is

$$\Phi_E = \int \vec{E} \cdot d\vec{S} = \frac{Q_{enc}}{\epsilon_0}$$

We can determine Q_{enc} by measuring the flux. To do that, we need to find $\vec{E}$ everywhere on a surface surrounding the box. $\vec{E}$ can be determined from $\vec{F} = q\vec{E}$ by placing a charge q on the surface and measuring the force it feels.

1c. To build this arrangement, we need to bring the two charges together from infinity. In doing so, we do work against the electric force,

$$\Delta U = k\frac{q_1 q_2}{r}$$

which cannot be zero. Therefore, these two charges can never have the same energy at distance r as at infinite separation.

2a. The electric field due to q_1 has components

$$E_{1x} = k\frac{q_1}{(a^2/2)}\cos(-45^o) = 5.97{\times}10^5\ N/C\ ,\ E_{1y} = k\frac{q_1}{(a^2/2)}\sin(-45^o) = -5.97{\times}10^5\ N/C$$

The electric field due to q_2 has components

$$E_{2x} = k\frac{q_2}{(a^2/2)}\cos(-135^o) = -7.16{\times}10^5\ N/C\ ,\ E_{2y} = k\frac{q_2}{(a^2/2)}\sin(-135^o) = -7.16{\times}10^5\ N/C$$

The electric field due to q_3 has components

$$E_{3x} = k\frac{q_3}{(a^2/2)}\cos(135^o) = -5.97\times10^5\ N/C\ ,\ E_{3y} = k\frac{q_3}{(a^2/2)}\sin(135^o) = 5.97\times10^5\ N/C$$

The electric field due to q_4 has components

$$E_{4x} = k\frac{q_4}{(a^2/2)}\cos(-45^o) = -7.16{\times}10^5\ N/C\ ,\ E_{4y} = k\frac{q_4}{(a^2/2)}\sin(-45^o) = -7.16{\times}10^5\ N/C$$

The total electric field has components

$$E_x = E_{1x} + E_{2x} + E_{3x} + E_{4x} = -1.432\times10^6\ N/C$$

$$E_y = E_{1y} + E_{2y} + E_{3y} + E_{4y} = -1.432\times10^6\ N/C$$

2b. The force on an electron has components

$$F_x = -eE_x = 2.29 \times 10^{-13}\ N$$

$$F_y = -eE_y = 2.29 \times 10^{-13}\ N$$

Its magnitude is $F = \sqrt{F_x^2 + F_y^2} = 3.24 \times 10^{-13}\ N$ and forms an angle of 45^o with the x-axis.

2c. The work is equal to the potential energy at P,

$$U = k\frac{(-e)q_1}{(a/\sqrt{2})} + k\frac{(-e)q_2}{(a/\sqrt{2})} + k\frac{(-e)q_3}{(a/\sqrt{2})} + k\frac{(-e)q_4}{(a/\sqrt{2})} = -7.6 \times 10^{-14}\ J$$

3a. At $r = a$, the surface charge density is

$$\sigma_a = \frac{Q_1}{4\pi a^2} = -6.6 \times 10^{-7} C/m^2$$

At $r = b$, the total charge must be $-Q_1$ in order to shield Q_1 and make $E = 0$ inside conducting shell. The surface charge density is

$$\sigma_b = \frac{-Q_1}{4\pi b^2} = +1.66 \times 10^{-7} C/m^2$$

The remaining charge $Q_2 - (-Q_1) = Q_2 + Q_1$ is at $r = c$. The surface charge density is

$$\sigma_c = \frac{Q_1 + Q_2}{4\pi c^2} = +9.8 \times 10^{-8} C/m^2$$

3b. Using Gauss's Law on a sphere of radius r, due to spherical symmetry, we have

$$E = \frac{Q_{enc}}{4\pi\epsilon_0 r^2}$$

For $r > c$, $Q_{enc} = Q_1 + Q_2$, $E = \frac{Q_1+Q_2}{4\pi\epsilon_0 r^2} = \frac{36{,}000\ N \cdot m^2/C}{r^2}$.

For $b < r < c$, $Q_{enc} = 0$, $E = 0\ N/C$.

For $a < r < b$, $Q_{enc} = Q_1$, $E = \frac{Q_1}{4\pi\epsilon_0 r^2} = -\frac{27{,}000\ N \cdot m^2/C}{r^2}$.

For $r < a$, $Q_{enc} = 0$, $E = 0\ N/C$.

3c. Using $V_1 - V_2 = \int_1^2 \vec{E} \cdot d\vec{l}$ along a radial path, we have:

For $r > c$,

$$V(r) = \int_r^\infty E(r')dr' = \int_r^\infty \frac{Q_1 + Q_2}{4\pi\epsilon_0 r'^2}\, dr' = \frac{Q_1 + Q_2}{4\pi\epsilon_0 r} = \frac{36,000\ V \cdot m}{r}$$

For $b < r < c$,

$$V(r) = V(c) + \int_r^c E(r')dr' = V(c) + 0 = \frac{Q_1 + Q_2}{4\pi\epsilon_0 c} = 20,000\ V$$

For $a < r < b$,

$$V(r) = V(b) + \int_r^b E(r')dr' = \frac{Q_1 + Q_2}{4\pi\epsilon_0 c} + \int_r^b \frac{Q_1}{4\pi\epsilon_0 r'^2} = \frac{Q_1 + Q_2}{4\pi\epsilon_0 c} - \frac{Q_1}{4\pi\epsilon_0 b} + \frac{Q_1}{4\pi\epsilon_0 r}$$

$$= 42,500\ V - \frac{27,000\ V \cdot m}{r}$$

For $r < a$,

$$V(r) = V(a) + \int_r^b E(r')dr' = V(a) + 0 = \frac{Q_1 + Q_2}{4\pi\epsilon_0 c} - \frac{Q_1}{4\pi\epsilon_0 b} + \frac{Q_1}{4\pi\epsilon_0 a} = -2,500\ V$$