PHYSICS 231 - Solution Key to Sample Test 2

1a. Initially, the capacitor has capacitance C_0, say. Then the voltage is $V_0 = Q/C_0$.

In the tank of oil, the capacitance becomes $C = KC_0$ and the voltage $V = Q/C = V_0/K$. Therefore, the voltage *decreases* and so does the electric field, because it is proportional to the voltage.

1b. The current is proportional to the speed v. Since the current is constant through the resistor, so is the speed v. The kinetic energy of an electron is $\frac{1}{2}mv^2$. If v doesn't change, no kinetic energy is lost.

As the electron goes through the resistor, it loses potential energy eV. The heat IV in the resistor is due to the loss of potential energy.

1c. The power is $P = V^2/R$, so $P \propto 1/R$. The 60-W bulb has *twice* the resistance of the 120-W bulb.

1d. Yes, if you shortcircuit the terminals of the battery.

2a. The capacitors between a and d are connected in parallel, so the capacitance between a and d is

$$C_{ad} = C_1 + C_2 = 20\ nF$$

The capacitors between b and d are connected in series, so the capacitance between b and d is

$$C_{bd} = \frac{1}{\frac{1}{C_3} + \frac{1}{C_4}} = 4.8\ nF$$

C_{ad} and C_{bd} are connected in series, so the capacitance between a and b is

$$C_{ab} = \frac{1}{\frac{1}{C_{ad}} + \frac{1}{C_{bd}}} = 3.87\ nF$$

2b. The charge on C_{ab} is

$$Q = C_{ab}V_{ab} = 1.16\ \mu C$$

This is also the charge on C_{ad} and C_{bd}, because they are connected in series.

C_{bd} is made of C_3 and C_4 in series, so C_3 and C_4 have charge Q (same as C_{bd}).

C_{ad} is made of C_1 and C_2 in parallel, so C_1 has charge Q_1 and C_2 has charge Q_2. They all have the same voltage, so

$$V_{ad} = \frac{Q}{C_{ad}} = \frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

so

$$Q_1 = C_1 \frac{Q}{C_{ad}} = 0.464\ \mu F \ , \quad Q_2 = C_2 \frac{Q}{C_{ad}} = 0.696\ \mu F$$

2c. $V_{ad} = Q/C_{ad} = 58$ V.

2d.

$$U_1 = \frac{Q_1^2}{2C_1} = 1.3 \times 10^{-5} J \ \ , \quad U_2 = \frac{Q_2^2}{2C_2} = 2.0 \times 10^{-5} J$$
$$U_3 = \frac{Q^2}{2C_3} = 8.4 \times 10^{-5} J \ \ , \quad U_4 = \frac{Q^2}{2C_4} = 5.6 \times 10^{-5} J$$

3a. The wire through c has two resistors in series and resistance

$$R_c = 2 + 4 = 6\,\Omega$$

The wire through d has two resistors in series and resistance

$$R_d = 4 + 2 = 6\,\Omega$$

R_c and R_d are in parallel, so the total resistance is

$$R = \frac{1}{\frac{1}{R_c} + \frac{1}{R_d}} = 3\,\Omega$$

3b. The current through a and b is

$$I_a = I_b = \frac{\mathcal{E}}{R} = 8\ A$$

Since $R_c = R_d$, this current splits into two equal currents,

$$I_c = I_d = \frac{8\ A}{2} = 4\ A$$

3c.

$$V_{ab} = \mathcal{E} = 24\ V$$
$$V_{cd} = V_{cb} + V_{bd} = 4I_c - 2I_d = 8\ V$$