PHYSICS 231 - Solution Key to Sample Test 1

1a. See FIGURE 22-22, p. 690.

1b. 1. FALSE: Gauss’s law is a general law of Nature.
     2. FALSE: There may be charges outside the surface.
     3. FALSE: It can be positive on parts of the surface and negative on other parts of the surface, as long as the TOTAL charge is zero.
     4. TRUE: Charges consist of electrons and protons.

1c. 3. The electron has negative charge. The force $F = -eE$ is in the direction opposite to $E$ and so is the acceleration, since $F = ma$.

2a. Charge $q_1$ creates an electric field of magnitude

$$E_1 = k\frac{q_1}{2^2} = 6.74 \times 10^3 \text{ N/C}$$

and direction $\vec{E}_1 = -6.74 \times 10^3 \hat{y}$.

Charge $q_2$ creates an electric field of magnitude

$$E_2 = k\frac{q_2}{2^2} = 11.24 \times 10^3 \text{ N/C}$$

and direction $\vec{E}_2 = -11.24 \times 10^3 \hat{y}$.

Charge $Q$ creates an electric field of magnitude

$$E_3 = k\frac{|Q|}{2^2 + 4^2} = 1.8 \times 10^3 \text{ N/C}$$

and forms an angle

$$\phi = \tan^{-1}\frac{2}{4} = 26.6^\circ$$

with the $x$-axis. Therefore,

$$\vec{E}_3 = 1.8 \times 10^3 \cos \phi \hat{x} + 1.8 \times 10^3 \sin \phi \hat{y} = 1.61 \times 10^3 \hat{x} + 0.8 \times 10^3 \hat{y}$$

The total electric field is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = 1.61 \times 10^3 \hat{x} + 5.3 \times 10^3 \hat{y}$$

Its magnitude is $E = 5.54 \times 10^3 \text{ N/C}$ and forms an angle

$$\theta = \tan^{-1}\frac{5.3 \times 10^3}{1.61 \times 10^3} = 73^\circ$$

with the $x$-axis.
2b. The force from charge \( q_1 \) has magnitude
\[
F_1 = k \frac{Q q_1}{4^2} = 6.74 \times 10^{-3} \text{ N}
\]
and direction \( \vec{F}_1 = -6.74 \times 10^{-3} \hat{x} \).

The force from charge \( q_2 \) has magnitude
\[
F_2 = k \frac{Q q_2}{4^2 + 4^2} = 5.6 \times 10^{-3} \text{ N}
\]
and direction
\[
\vec{F}_2 = -5.6 \times 10^{-3} (\cos 45^\circ \hat{x} + \sin 45^\circ \hat{y}) = -4 \times 10^{-3} (\hat{x} + \hat{y})
\]

The total force is
\[
\vec{F} = \vec{F}_1 + \vec{F}_2 = -10.74 \times 10^{-3} \hat{x} - 4 \times 10^{-3} \hat{y}
\]

3a. \( \sigma_1 = \frac{q_1}{A} = 1.33 \mu \text{C/m}^2 \), \( \sigma_2 = \frac{q_2}{A} = 3.33 \mu \text{C/m}^2 \), \( \sigma_3 = \frac{q_3}{A} = -4.67 \mu \text{C/m}^2 \)

3b. At \( P \),
\[
E = \frac{\sigma_1}{2\epsilon_0} - \frac{\sigma_2}{2\epsilon_0} - \frac{\sigma_3}{2\epsilon_0} = \frac{1.33 - 3.33 + 4.67}{2\epsilon_0} \times 10^{-6} = 1.51 \times 10^5 \text{ N/C}
\]
At \( Q \),
\[
E = \frac{\sigma_1}{2\epsilon_0} + \frac{\sigma_2}{2\epsilon_0} - \frac{\sigma_3}{2\epsilon_0} = \frac{1.33 + 3.33 + 4.67}{2\epsilon_0} \times 10^{-6} = 5.27 \times 10^5 \text{ N/C}
\]
At \( R \),
\[
E = \frac{\sigma_1}{2\epsilon_0} + \frac{\sigma_2}{2\epsilon_0} + \frac{\sigma_3}{2\epsilon_0} = \frac{1.33 + 3.33 - 4.67}{2\epsilon_0} \times 10^{-6} = 0 \text{ N/C}
\]

3c. \( F = q_3 (E_1 + E_2) = q_3 \frac{\sigma_1 + \sigma_2}{2\epsilon_0} = -1.9 \text{ N} \)