PHYSICS 231 - Solution Key to Sample Test 1

1a. See FIGURE 22-22, p. 690.

1b.
1. FALSE: Gauss's law is a general law of Nature.
2. FALSE: There may be charges outside the surface.
3. FALSE: It can be positive on parts of the surface and negative on other parts of the surface, as long as the TOTAL charge is zero.
4. TRUE: Charges consist of electrons and protons.

1c. $\boxed{3.}$ The electron has negative charge. The force $F = -eE$ is in the direction opposite to E and so is the acceleration, since $F = ma$.

2a. Charge q_1 creates an electric field of magnitude

$$E_1 = k\frac{q_1}{2^2} = 6.74 \times 10^3 \ N/C$$

and direction $\vec{E}_1 = -6.74 \ \times 10^3 \hat{y}$.

Charge q_2 creates an electric field of magnitude

$$E_2 = k\frac{q_2}{2^2} = 11.24 \ \times 10^3 N/C$$

and direction $\vec{E}_2 = -11.24 \ \times 10^3 \hat{y}$.

Charge Q creates an electric field of magnitude

$$E_3 = k\frac{|Q|}{2^2 + 4^2} = 1.8 \ \times 10^3 N/C$$

and forms an angle

$$\phi = \tan^{-1}\frac{2}{4} = 26.6^o$$

with the x-axis. Therefore,

$$\vec{E}_3 = 1.8 \times 10^3 \cos\phi \hat{x} + 1.8 \times 10^3 \sin\phi \hat{y} = 1.61 \times 10^3 \hat{x} + 0.8 \times 10^3 \hat{y}$$

The total electric field is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = 1.61 \times 10^3 \hat{x} + 5.3 \times 10^3 \hat{y}$$

Its magnitude is $E = 5.54 \times 10^3$ N/C and forms an angle

$$\theta = \tan^{-1}\frac{5.3 \times 10^3}{1.61 \times 10^3} = 73^o$$

with the x-axis.

2b. The force from charge q_1 has magnitude

$$F_1 = k\frac{Qq_1}{4^2} = 6.74 \times 10^{-3}\ N$$

and direction $\vec{F}_1 = -6.74 \times 10^{-3}\hat{x}$.

The force from charge q_2 has magnitude

$$F_2 = k\frac{Qq_2}{4^2 + 4^2} = 5.6 \times 10^{-3}\ N$$

and direction

$$\vec{F}_2 = -5.6 \times 10^{-3}(\cos 45^o\ \hat{x} + \sin 45^o\ \hat{y}) = -4 \times 10^{-3}(\hat{x} + \hat{y})$$

The total force is

$$\vec{F} = \vec{F}_1 + \vec{F}_2 = -10.74 \times 10^{-3}\ \hat{x} - 4 \times 10^{-3}\ \hat{y}$$

3a.

$$\sigma_1 = \frac{q_1}{A} = 1.33\ \mu C/m^2\ \ , \quad \sigma_2 = \frac{q_2}{A} = 3.33\ \mu C/m^2\ \ , \quad \sigma_3 = \frac{q_3}{A} = -4.67\ \mu C/m^2$$

3b. At P,

$$E = \frac{\sigma_1}{2\epsilon_0} - \frac{\sigma_2}{2\epsilon_0} - \frac{\sigma_3}{2\epsilon_0} = \frac{1.33 - 3.33 + 4.67}{2\epsilon_0} \times 10^{-6} = 1.51 \times 10^5\ N/C$$

At Q,

$$E = \frac{\sigma_1}{2\epsilon_0} + \frac{\sigma_2}{2\epsilon_0} - \frac{\sigma_3}{2\epsilon_0} = \frac{1.33 + 3.33 + 4.67}{2\epsilon_0} \times 10^{-6} = 5.27 \times 10^5\ N/C$$

At R,

$$E = \frac{\sigma_1}{2\epsilon_0} + \frac{\sigma_2}{2\epsilon_0} + \frac{\sigma_3}{2\epsilon_0} = \frac{1.33 + 3.33 - 4.67}{2\epsilon_0} \times 10^{-6} = 0\ N/C$$

3c.

$$F = q_3(E_1 + E_2) = q_3\frac{\sigma_1 + \sigma_2}{2\epsilon_0} = -1.9\ N$$