PHYSICS 231 - Solution Key to the Sample Final Exam

1a. No. By Gauss's Law,

$$\Phi_E = \frac{Q_{enc}}{\epsilon_0}$$

regardless of the shape of the surface.

1b. Inserting the dielectric increases the capacitance ($C = KC_0 > C_0$). The voltage remains the same, because we keep the capacitor connected to the battery.

The charge increases:

$$Q' = CV = KC_0V = KQ > Q$$

and so does the energy:

$$U' = \frac{1}{2}CV^2 = \frac{1}{2}KC_0V^2 = KU > U$$

1c. Let V_{ab} be the voltage of the terminals. To make $V_{ab} < 0$, connect the battery in series to another battery of emf $\mathcal{E}'$ and negligible internal resistance with positive terminals connected to each other. Then $V_{ab} = -\mathcal{E}'$.

1d. The power is $P = V^2/R$, so the 25 W bulb has bigger resistance. If they are connected in series, they share the same current, so each gets power $P = I^2R$. The 25 W bulb gets more power (8 times more) than the 200 W bulb and burns.

1e. We have

$$L \propto \Phi \propto A \propto d^2$$

Therefore,

$$\frac{L_1}{L_2} = \left(\frac{d_1}{d_2}\right)^2 = 2^2 = 4$$

1f. The electrons oscillate in the wire and never go anywhere. However, the power is not transmitted by the electrons. In a circuit the electrons feel an electric force because of the potential, regardless of where they are, and as a result they get kinetic energy. They move very slowly, but they start to move almost instantaneously after the switch is closed. The power they get travels to them from the source through the wire at about the speed of light.

2a. From Gauss's Law, because of spherical symmetry,

$$\vec{E} = E\hat{r} \ , \quad E = \frac{Q_{enc}}{4\pi\epsilon_0 r^2} = \frac{kQ_{enc}}{r^2}$$

At $r = 6$ cm, $Q_{enc} = q_1$, so

$$E = \frac{kq_1}{r^2} = -\frac{8.988 \times 10^9 \times 5 \times 10^{-9}}{0.06^2} = -1.25 \times 10^4 \ N/C$$

At $r = 18$ cm, we are inside the conductor, so

$$E = 0 \ N/C$$

At $r = 90$ cm, $Q_{enc} = q_1 + q_2$, so

$$E = \frac{kq_1}{r^2} = \frac{8.988 \times 10^9 \times (-5+4) \times 10^{-9}}{0.90^2} = -11\ N/C$$

2b. To make $E = 0$ in the shell, we need charge $-q_1 = +5$ nC on the inner surface of the shell (at $r = r_2$). The remainder, $q_2 - (-q_1) = 4 - 5 = -1$ nC resides on the outer surface of the shell (at $r = r_3$).

3. Consider the two points on the coil on which the $\vec{B}$-field is shown in the figure. $\vec{B}$ has two components. The y-components give rise to two forces which cancel each other (because the current is in opposite directions at the two points).

B_x creates a force in the y-direction. As we move along the wire, the current remains perpendicular to the magnetic field, so the total force is

$$F = ILB_x$$

where L is the total length of the wire,

$$L = N\pi d = 50 \times \pi \times 0.02 = 3.14\ m$$

Also $B_x = B \sin\theta$, so

$$F = ILB \sin\theta = 1.5 \times 3.14 \times 0.3 \sin 50^o = 1.08\ N$$

in the y-direction.

4a. We have an $R - L$ circuit of total resistance $R_0 + R$, so

$$I = \frac{\mathcal{E}}{R_0 + R}\left(1 - e^{-t/\tau}\right) \quad , \quad \tau = \frac{L}{R_0 + R}$$

(i) At $t = 0$, $I = 0$ A.

(ii) At $t = 0.05$ s,

$$I = \frac{40}{60 + 200}\left(1 - e^{-\frac{60+200}{5} \times 0.05}\right) = 0.142\ A$$

(iii) At $t \to \infty$,

$$I = \frac{\mathcal{E}}{R_0 + R} = 0.154\ A$$

4b. The voltage across R_0 is $V_{ac} = \mathcal{E}$, so the current through R_0 is

$$I = \frac{\mathcal{E}}{R_0} = \frac{40}{60} = 0.667\ A$$

at all times.

Since b and c are short-circuited, R and L form a circuit with current

$$I_1 = I_0 e^{-Rt/L}$$

where $I_0 = 0.154$ A (found above).

At the junction c, $I = I_1 + I_2$, where I_2 is the current through switch S_2.

(i) At $t = 0$, $I = 0.667$ A, $I_1 = 0.154$ A and $I_2 = I - I_1 = 0.513$ A.

(ii) At $t = 0.1$ s, $I = 0.667$ A,

$$I_1 = 0.154e^{-\frac{200}{5}\times 0.1} = 0.003\ A$$

and $I_2 = 0.667 - 0.003 = 0.664$ A.

(iii) At $t \to \infty$, $I = 0.667$ A, $I_1 = 0$ and $I_2 = I = 0.667$ A.

5a. We are given $V_R = 120$ V and

$$\omega = 2\pi f = 2\pi \times 60 = 377\ rad/s$$

The amplitude of the current is

$$I_0 = \frac{V_R}{R} = \frac{120}{50} = 2.4\ A$$

The current is $I = I_0 \cos(\omega t)$.

5b.

$$X_L = \omega L = 377 \times 0.1 = 37.7\ \Omega$$

5c. For the inductor, the amplitude of the voltage is

$$V_{L0} = I_0 X_L = 2.4 \times 37.7 = 90\ V$$

We also have $\phi = 90^o$, so

$$V_L = V_{L0} \cos\left(\omega t + \frac{\pi}{2}\right) = -V_{L0} \sin(\omega t) = -(90\ V) \sin[(377\ rad/s)t]$$