PHYSICS 231 - Solution Key to the Final Exam

1a. If you connect N bulbs each of resistance R in series to a battery of emf $\mathcal{E}$, the current is

$$I = \frac{\mathcal{E}}{NR}$$

The power in each bulb is

$$P = I^2 R = \frac{\mathcal{E}^2}{N^2 R}$$

As N increases, P decreases, so each bulb gets dimmer.

1b. The resistance is

$$R = \frac{\rho L}{A}$$

The power is $P = \mathcal{E}^2/R$. With a larger $\mathcal{E}$, we can use larger R. Since $R \propto 1/A$, larger R means smaller A, so thinner wire.

1c. No. In fact, the particle can move along the field lines. This requires no force, because $\vec{F} = q\vec{v} \times \vec{B} = \vec{0}$, if $\vec{v}$ is along $\vec{B}$.

1d. We have

$$\tan\phi = \frac{X_L - X_C}{R}$$

At resonance, $X_L = X_C$. If we are far from resonance, $X_L - X_C$ is not small. If R is small, then $\tan\phi \to \infty$, so

$$\phi \approx 90^o$$

The power factor is $\cos\phi \approx \cos 90^0 = 0$. Very little power is lost.

2a. The electric field due to q_1 is

$$\vec{E}_1 = \frac{kq_1}{d_1^2}\,\hat{x}$$

The electric field due to q_2 is

$$\vec{E}_2 = \frac{kq_2}{d_2^2}\,(-\hat{x})$$

The total electric field is

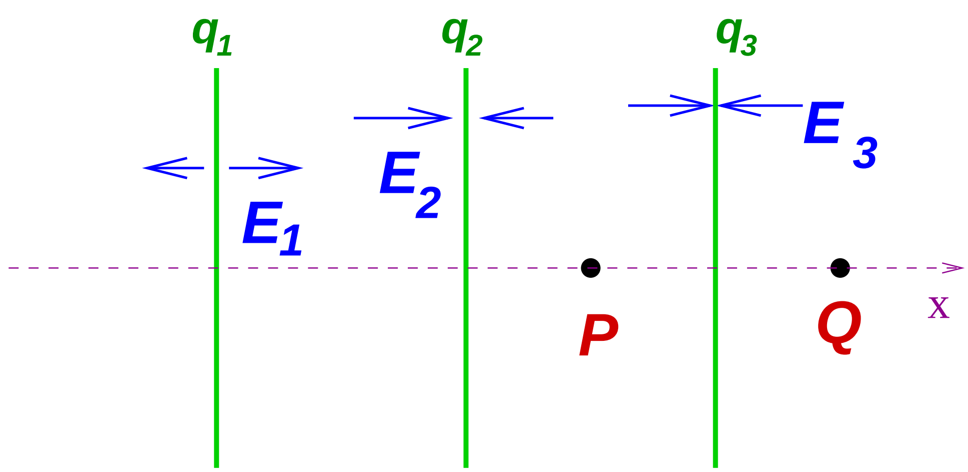
$$\vec{E} = \vec{E}_1 + \vec{E}_2 = 8.988 \times 10^9 \left(\frac{2.5 \times 10^{-6}}{3.5^2} + \frac{1.5 \times 10^{-6}}{0.9^2} \right) \hat{x} = (1.8 \times 10^4\ N/C)\,\hat{x}$$

2b. The force is

$$\vec{F} = (-e)\vec{E} = -1.6 \times 10^{-19} \times 1.8 \times 10^4\,\hat{x} = -(3 \times 10^{-15}\ N)\,\hat{x}$$

2c. The work done is equal to the potential energy of the electron,

$$\begin{aligned} U &= k\frac{(-e)q_1}{d_1} + k\frac{(-e)q_2}{d_2} \\ &= 8.988\times 10^9 \times (-1.6\times 10^{-19})\left(\frac{2.5\times 10^{-6}}{3.5} + \frac{-1.5\times 10^{-6}}{0.9}\right) \\ &= 1.37\times 10^{-15}\ J \end{aligned}$$



3a.

$$\sigma_1 = \frac{q_1}{A} = 1.7\ \mu C/m^2\ ,\quad \sigma_2 = \frac{q_2}{A} = -1.1\ \mu C/m^2\ ,\quad \sigma_3 = \frac{q_3}{A} = -0.6\ \mu C/m^2$$

3b. Magnitude of electric field due to each sheet:

$$E_1 = \frac{|\sigma_1|}{2\epsilon_0} = 9.7\times 10^4\ N/C\ ,\quad E_2 = \frac{|\sigma_2|}{2\epsilon_0} = 6.5\times 10^4\ N/C\ ,\quad E_3 = \frac{|\sigma_3|}{2\epsilon_0} = 3.2\times 10^4\ N/C$$

Direction as shown.

At P,

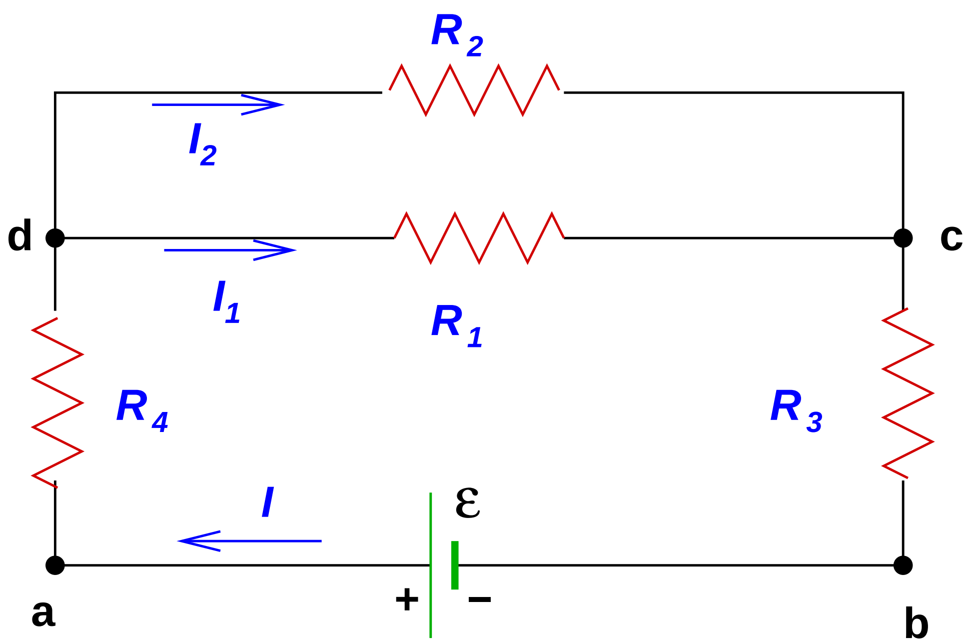
$$\vec{E} = (E_1 - E_2 + E_3)\,\hat{x} = (6.4\times 10^4\ N/C)\,\hat{x}$$

At Q,

$$\vec{E} = (E_1 - E_2 - E_3)\,\hat{x} = \vec{0}$$

3c.

$$\vec{F} = q_2(E_1\vec{x} + E_3\vec{x}) = (-0.5\ N)\,\hat{x}$$



4a. R_1 and R_2 are in parallel, so the resistance between c and d is

$$R_{cd} = \frac{1}{1/R_1 + 1/R_2} = \frac{1}{1/4 + 1/2} = 1.33\ \Omega$$

The total resistance is

$$R = R_4 + R_{cd} + R_3 = 4 + 1.33 + 2 = 7.33\ \Omega$$

The current I is

$$I = \frac{\mathcal{E}}{R} = \frac{24}{7.33} = 3.27\ A$$

At the junction d it splits,

$$I = I_1 + I_2$$

We have

$$V_{dc} = I_1 R_1 = I_2 R_2 = I R_{cd}$$

so

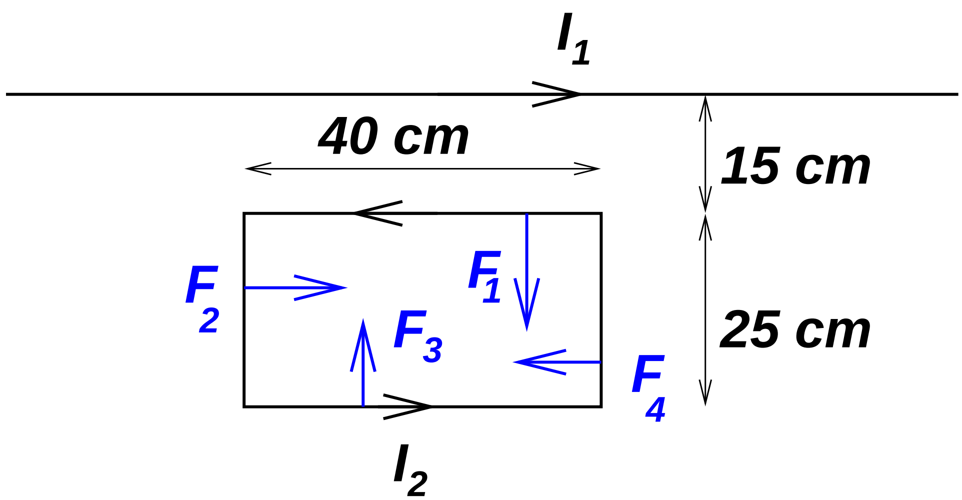
$$I_1 = I \frac{R_{cd}}{R_1} = 3.27 \times \frac{1.33}{4} = 1.09\ A \ \ , \quad I_2 = I - I_1 = 2.18\ A$$

4b.

$$V_{cd} = -I_1 R_1 = -1.09 \times 4 = -4.36\ V$$

4c.

$$P = I^2 R = (3.27)^2 \times 7.33 = 78\ W$$



5. Magnetic field due to straight wire:

$$B(r) = \frac{\mu_0 I_1}{2\pi r}$$

Magnitudes of forces:

$$F_1 = I_2 L B(r_1) \ , \quad F_2 = I_2 L B(r_2)$$

where $L = 40$ cm and $r_1 = 15$ cm, $r_2 = 15 + 25 = 40$ cm.

The other two forces, F_2 and F_4 are harder to calculate but they cancel each other. Since $F_1 > F_2$, the net force has magnitude

$$F = F_1 - F_2 = \frac{\mu_0 L I_1 I_2}{2\pi r_1} - \frac{\mu_0 L I_1 I_2}{2\pi r_2} = \frac{4\pi \times 10^{-7} \times 0.40 \times 3 \times 1.8}{2\pi} \left(\frac{1}{0.15} - \frac{1}{0.40} \right) = 1.8 \times 10^{-6} \ N$$

and direction same as $\vec{F}_1$.

6a.

$$I = \frac{\mathcal{E}}{R} \left(1 - e^{-t/\tau}\right) \ , \quad \tau = \frac{L}{R}$$

At $t = 0$ s, $I = 0$ A.

At $t = 0.05$ s,

$$I = \frac{40}{200} \left(1 - e^{-200 \times 0.05/5}\right) = 0.17 \ A$$

At $t \to \infty$,

$$I = \frac{\mathcal{E}}{R} = \frac{40}{200} = 0.2 \ A$$

6b. $U = \frac{1}{2} L I^2$

At $t = 0$ s, $U = 0$ J.

At $t = 0.05$ s,

$$U = \frac{1}{2} \times 5 \times (0.17)^2 = 0.07 \ J$$

At $t \to \infty$,

$$U = \frac{1}{2} \times 5 \times (0.2)^2 = 0.1 \ J$$