

PHYSICS 231 – FINAL EXAM

NAME:

STUDENT ID #:

USEFUL CONSTANTS

$$\epsilon_0 = 8.85 \times 10^{-12} \text{C}^2 / (\text{N} \cdot \text{m}^2)$$

$$k = \frac{1}{4\pi\epsilon_0} = 8.988 \times 10^9 \text{N} \cdot \text{m}^2 / \text{C}^2$$

$$e = 1.6 \times 10^{-19} \text{C}$$

$$m_e = 9.1 \times 10^{-31} \text{kg}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{T} \cdot \text{m} / \text{A}$$

$$g = 9.81 \text{N} / \text{kg}$$

USEFUL FORMULAS

Coulomb's law:

$$F = k \frac{|q_1 q_2|}{r^2}$$

Electric field:

$$\vec{E} = \frac{\vec{F}}{q}$$

Electric dipole:

- dipole moment: $p = qd$

- torque: $\vec{\tau} = \vec{p} \times \vec{E}$

- energy: $U = -\vec{p} \cdot \vec{E}$

Electric flux:

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

Gauss's law:

$$\Phi_E = \frac{Q_{enc}}{\epsilon_0}$$

Electric field

► due to infinite wire:

$$E = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{r}$$

► due to infinite sheet:

$$E = \frac{\sigma}{2\epsilon_0}$$

Potential energy:

$$U = k \frac{q_1 q_2}{r}$$

Potential:

$$V = \frac{U}{q}$$

Potential difference:

$$V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l}$$

Electric field:

$$\vec{E} = -\vec{\nabla}V$$

Capacitance:

$$C = \frac{Q}{V}$$

Parallel-plate capacitor:

$$C = \epsilon_0 \frac{A}{d}$$

Capacitors in series:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Capacitors in parallel:

$$C_{eq} = C_1 + C_2$$

Energy:

$$U = \frac{Q^2}{2C} = \frac{1}{2} CV^2 = \frac{1}{2} QV$$

Energy density:

$$u = \frac{1}{2} \epsilon_0 E^2$$

Capacitor in dielectric:

$$C = KC_0$$

Energy density in dielectric:

$$u = \frac{1}{2} \epsilon E^2, \quad \epsilon = K\epsilon_0$$

Current

$$I = \frac{dQ}{dt} = n|q|v_d A$$

Ohm's law:

$$V = IR, \quad R = \frac{\rho L}{A}$$

Variation with temperature:

$$\rho(T) = \rho_0 [1 + \alpha(T - T_0)]$$

Power into a resistor:

$$P = VI = I^2 R = \frac{V^2}{R}$$

Resistors in series:

$$R_{eq} = R_1 + R_2$$

Resistors in parallel:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Kirchhoff's rules:

$$\sum I = 0, \quad \sum V = 0$$

RC circuit

► capacitor charging:

$$Q = C\mathcal{E} \left(1 - e^{-\frac{t}{RC}}\right), \quad I = \frac{dQ}{dt}$$

► capacitor discharging:

$$Q = Q_0 e^{-\frac{t}{RC}}, \quad I = \frac{dQ}{dt}$$

Magnetic force

► on point charge: $\vec{F} = q\vec{v} \times \vec{B}$

► on wire: $\vec{F} = I \int d\vec{l} \times \vec{B}$

Magnetic flux (closed surface):

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = 0$$

In uniform \vec{B} , charge on circle

$$R = \frac{mv}{|q|B}$$

Current loop:

$$\vec{\mu} = I\vec{A}$$

► force: $\vec{F} = \vec{0}$.

► torque: $\vec{\tau} = \vec{\mu} \times \vec{B}$.

► energy: $U = -\vec{\mu} \cdot \vec{B}$.

Magnetic field

► due to point charge:

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \vec{r}}{r^3}$$

► due to wire:

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times \vec{r}}{r^3}$$

► Ampère's law:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

► outside infinite straight wire:

$$B = \frac{\mu_0 I}{2\pi r}$$

► on axis of circular wire:

$$B_x = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}}$$

► at center of N circular loops:

$$B_x = \frac{\mu_0 N I}{2a}$$

► inside solenoid:

$$B = \mu_0 n I$$

Emf

► in closed loop:

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

► in moving loop:

$$\mathcal{E} = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

► in uniform $\vec{B} \perp \vec{L} \perp \vec{v}$:

$$\mathcal{E} = vBL$$

Mutual inductance:

$$M = \frac{N_2 \Phi_{B2}}{I_1} = \frac{N_1 \Phi_{B1}}{I_2}$$

$$\mathcal{E}_2 = -M \frac{dI_1}{dt}, \quad \mathcal{E}_1 = -M \frac{dI_2}{dt}$$

Self-inductance:

$$L = \frac{N\Phi_B}{I}, \quad \mathcal{E} = -L \frac{dI}{dt}$$

Energy:

$$U = \frac{1}{2} L I^2$$

Energy density:

$$u = \frac{B^2}{2\mu_0}$$

$R - L$ circuit:

$$I = \frac{\mathcal{E}}{R} (1 - e^{-t/\tau}), \quad \tau = \frac{L}{R}$$

$L - C$ circuit:

$$\omega = \frac{1}{\sqrt{LC}}$$

$L - R - C$ circuit:

$$\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

AC current and voltage

$$I = I_0 \cos(\omega t), \quad V = V_0 \cos(\omega t + \phi)$$

► resistor (resistance R):

$$\phi = 0, \quad V_0 = I_0 R$$

► inductor:

$$\phi = +90^\circ, \quad V_0 = I_0 X_L$$

inductive reactance $X_L = \omega L$.

► capacitor:

$$\phi = -90^\circ, \quad V_0 = I_0 X_C$$

capacitive reactance $X_C = \frac{1}{\omega C}$.

► $R - L - C$ circuit:

$$\tan \phi = \frac{X_L - X_C}{R}, \quad V_0 = I_0 Z$$

impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$

Rectified average:

$$I_{rav} = \frac{2}{\pi} I_0$$

Root-mean-square:

$$I_{rms} = \frac{I_0}{\sqrt{2}}, \quad V_{rms} = \frac{V_0}{\sqrt{2}}$$

Average power:

$$P_{av} = \frac{1}{2} V_0 I_0 \cos \phi = V_{rms} I_{rms} \cos \phi$$

Transformer:

$$\frac{V_{02}}{V_{01}} = \frac{N_2}{N_1}, \quad V_{01} I_{01} = V_{02} I_{02}$$