PHYSICS 231 – FINAL EXAM

NAME:

STUDENT ID #:

USEFUL CONSTANTS

$$\epsilon_0 = 8.85 \times 10^{-12} C^2/(N \cdot m^2)$$

$$k = \frac{1}{4\pi\epsilon_0} = 8.988 \times 10^9 \ N \cdot m^2/C^2$$

$$e = 1.6 \times 10^{-19} C$$

$$m_e = 9.1 \times 10^{-31} kg$$

$$\mu_0 = 4\pi \times 10^{-7} \ T \cdot m/A$$

$$g = 9.81 \ N/kg$$

USEFUL FORMULAS

Coulomb's law:

$$F = k \frac{|q_1 q_2|}{r^2}$$

Electric field:

$$\vec{E} = \frac{\vec{F}}{q}$$

Electric dipole:

- dipole moment: $p = qd$
- torque: $\vec{\tau} = \vec{p} \times \vec{E}$
- energy: $U = -\vec{p} \cdot \vec{E}$

Electric flux:

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

Gauss's law:

$$\Phi_E = \frac{Q_{enc}}{\epsilon_0}$$

Electric field

▶ due to infinite wire:

$$E = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{r}$$

▶ due to infinite sheet:

$$E = \frac{\sigma}{2\epsilon_0}$$

Potential energy:

$$U = k \frac{q_1 q_2}{r}$$

Potential:

$$V = \frac{U}{q}$$

Potential difference:

$$V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l}$$

Electric field:

$$\vec{E} = -\vec{\nabla} V$$

Capacitance:

$$C = \frac{Q}{V}$$

Parallel-plate capacitor:

$$C = \epsilon_0 \frac{A}{d}$$

Capacitors in series:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Capacitors in parallel:

$$C_{eq} = C_1 + C_2$$

Energy:

$$U = \frac{Q^2}{2C} = \frac{1}{2} C V^2 = \frac{1}{2} Q V$$

Energy density:

$$u = \frac{1}{2} \epsilon_0 E^2$$

Capacitor in dielectric:

$$C = K C_0$$

Energy density in dielectric:

$$u = \frac{1}{2} \epsilon E^2 \ , \quad \epsilon = K \epsilon_0$$

Current

$$I = \frac{dQ}{dt} = n|q| v_d A$$

Ohm's law:

$$V = IR \ , \qquad R = \frac{\rho L}{A}$$

Variation with temperature:

$$\rho(T) = \rho_0 [1 + \alpha (T - T_0)]$$

Power into a resistor:

$$P = VI = I^2 R = \frac{V^2}{R}$$

Resistors in series:

$$R_{eq} = R_1 + R_2$$

Resistors in parallel:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Kirchhoff's rules:

$$\sum I = 0 \ , \qquad \sum V = 0$$

RC circuit

▶ capacitor charging:

$$Q = C\mathcal{E}\left(1 - e^{-\frac{t}{RC}}\right) \quad , \quad I = \frac{dQ}{dt}$$

▶ capacitor discharging:

$$Q = Q_0 e^{-\frac{t}{RC}} \quad , \quad I = \frac{dQ}{dt}$$

Magnetic force

▶ on point charge: $\vec{F} = q\vec{v} \times \vec{B}$

▶ on wire: $\vec{F} = I \int d\vec{l} \times \vec{B}$

Magnetic flux (closed surface):

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = 0$$

In uniform $\vec{B}$, charge on circle

$$R = \frac{mv}{|q|B}$$

Current loop:

$$\vec{\mu} = I\vec{A}$$

▶ force: $\vec{F} = \vec{0}$.

▶ torque: $\vec{\tau} = \vec{\mu} \times \vec{B}$.

▶ energy: $U = -\vec{\mu} \cdot \vec{B}$.

Magnetic field

▶ due to point charge:

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \vec{r}}{r^3}$$

▶ due to wire:

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times \vec{r}}{r^3}$$

▶ Ampère's law:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

▶ outside infinite straight wire:

$$B = \frac{\mu_0 I}{2\pi r}$$

▶ on axis of circular wire:

$$B_x = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}}$$

▶ at center of N circular loops:

$$B_x = \frac{\mu_0 N I}{2a}$$

▶ inside solenoid:

$$B = \mu_0 n I$$

Emf

▶ in closed loop:

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

▶ in moving loop:

$$\mathcal{E} = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

▶ in uniform $\vec{B} \perp \vec{L} \perp \vec{v}$:

$$\mathcal{E} = vBL$$

Mutual inductance:

$$M = \frac{N_2 \Phi_{B2}}{I_1} = \frac{N_1 \Phi_{B1}}{I_2}$$

$$\mathcal{E}_2 = -M\frac{dI_1}{dt} \quad , \quad \mathcal{E}_1 = -M\frac{dI_2}{dt}$$

Self-inductance:

$$L = \frac{N\Phi_B}{I} \quad , \quad \mathcal{E} = -L\frac{dI}{dt}$$

Energy:

$$U = \frac{1}{2}LI^2$$

Energy density:

$$u = \frac{B^2}{2\mu_0}$$

$R - L$ circuit:

$$I = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right) \quad , \quad \tau = \frac{L}{R}$$

$L - C$ circuit:

$$\omega = \frac{1}{\sqrt{LC}}$$

$L - R - C$ circuit:

$$\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

AC current and voltage

$I = I_0 \cos(\omega t)$, $V = V_0 \cos(\omega t + \phi)$

▶ resistor (resistance R):

$$\phi = 0 \quad , \quad V_0 = I_0 R$$

▶ inductor:

$$\phi = +90^o \quad , \quad V_0 = I_0 X_L$$

inductive reactance $X_L = \omega L$.

▶ capacitor:

$$\phi = -90^o \quad , \quad V_0 = I_0 X_C$$

capacitive reactance $X_C = \frac{1}{\omega C}$.

▶ $R - L - C$ circuit:

$$\tan\phi = \frac{X_L - X_C}{R} \quad , \quad V_0 = I_0 Z$$

impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$

Rectified average:

$$I_{rav} = \frac{2}{\pi} I_0$$

Root-mean-square:

$$I_{rms} = \frac{I_0}{\sqrt{2}} \quad , \quad V_{rms} = \frac{V_0}{\sqrt{2}}$$

Average power:

$$P_{av} = \frac{1}{2} V_0 I_0 \cos\phi = V_{rms} I_{rms} \cos\phi$$

Transformer:

$$\frac{V_{02}}{V_{01}} = \frac{N_2}{N_1} \quad , \quad V_{01} I_{01} = V_{02} I_{02}$$

Problem 1

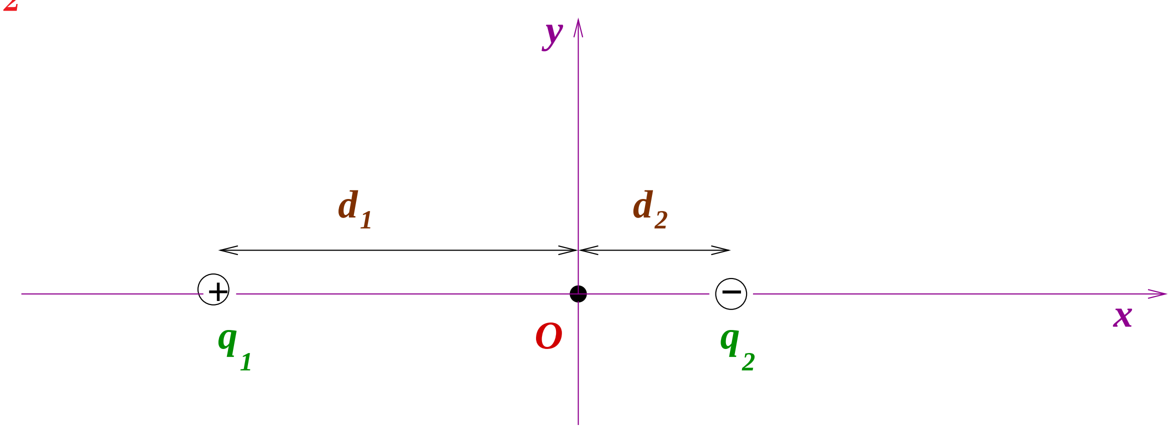
(a) You connect a number of identical light bulbs to a flashlight battery. What happens to the brightness of each bulb as more and more bulbs are added to the circuit if you connect them in series? Explain.

(b) Small aircraft often have 24-V electrical systems rather than the 12-V systems in automobiles. The explanation given by aircraft designers is that a 24-V system weighs less than a 12-V system, because thinner wires can be used. Explain why this is so.

(c) The magnetic force on a moving charged particle is always perpendicular to the magnetic field. Is the trajectory of a moving charged particle always perpendicular to the magnetic field lines? If so, explain why. If not, give an example to justify your answer.

(d) In a series $L-R-C$ AC circuit, what are the phase angle ϕ and power factor $\cos\phi$ when the resistance is much smaller than the inductive or capacitive reactance and the circuit is operated far from resonance? Explain.

Problem 2



In the figure, the charges are $q_1 = +2.5\ \mu C$ and $q_2 = -1.5\ \mu C$, at distances $d_1 = 3.5$ m and $d_2 = 0.9$ m from the origin O, as shown.

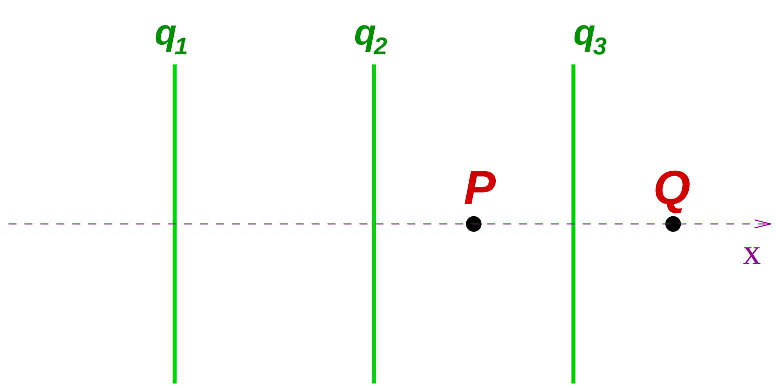
(a) What is the electric field at O (magnitude and direction)?

(b) Calculate the force on an electron at O (magnitude and direction).

(c) How much work do you have to do to bring the electron from infinity to O?

Problem 3

Three large parallel sheets have charges $q_1 = +6\ \mu C$, $q_2 = -4\ \mu C$, and $q_3 = -2\ \mu C$. Each sheet has area $A = 3.5\ m^2$.



(a) What is the charge density on each of the three sheets?

(b) Find the magnitude and direction of the electric field at points P and Q.

(c) What is the force exerted on the middle sheet by the other two sheets?

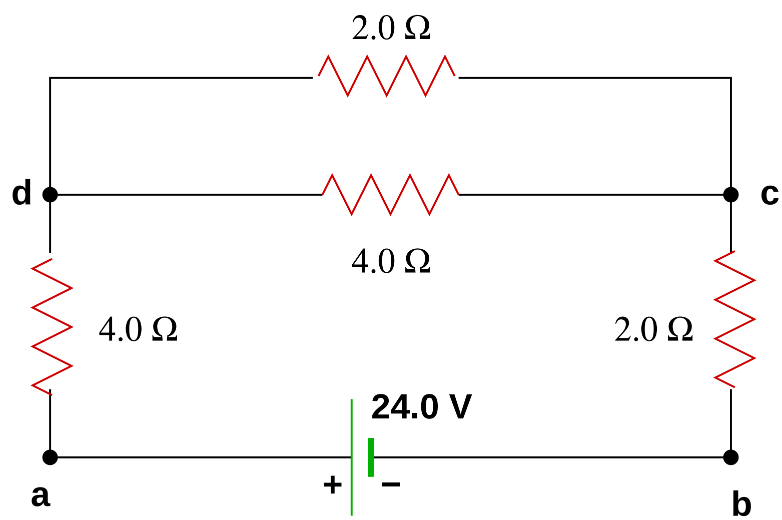
Problem 4

In the circuit shown

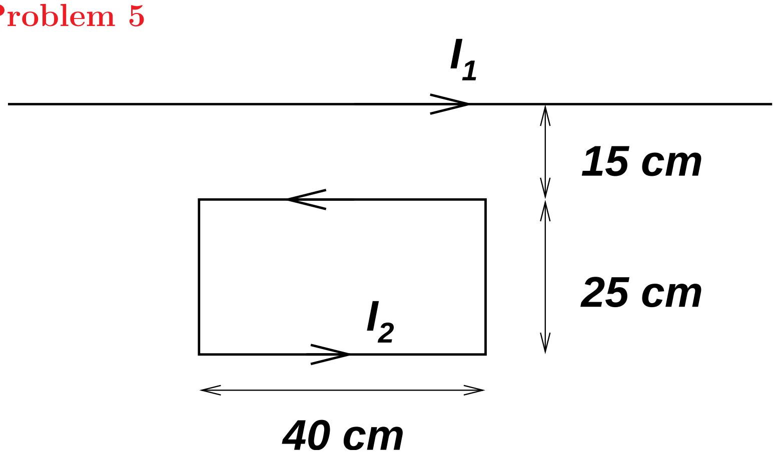
(a) Calculate the current through each of the four resistors.

(b) What is the voltage V_{cd}?

(c) What is the power dissipated in the circuit?



Problem 5



A long straight wire carries a current $I_1 = 3$ A. A 40 cm × 25 cm rectangular loop carrying a current $I_2 = 1.8$ A is placed a distance 15 cm away from the wire, as shown.

- Find the magnitude and direction of the net force on the loop.

Problem 6

In the circuit shown, $R = 200\ \Omega$, $L = 5$ H, and $\mathcal{E} = 40$ V.

(a) What is the current, *(i)* right after, *(ii)* at 0.05 s after and *(iii)* a very long time after switch S is closed?

(b) What is the energy stored in the inductor, *(i)* right after, *(ii)* at 0.05 s after and *(iii)* a very long time after switch S is closed?