LECTURE 4

Black Branes from String Theory

The Kaluza-Klein picture we have presented above is classical and, just as we need quantum mechanics to understand the electron (which is a point particle, i.e. a singularity), we need quantum mechanics to understand a black hole. Unfortunately (because it is very complicated) the only quantum theory of gravity we possess is string theory. To avoid infinities in the construction of quantum gravity, string theory complicates things:

- Strings live in ten dimensions
- Strings vibrate. The greater the frequency, the higher the mass. Also an infinite number of vibrational modes should correspond to an infinite number of particles. The massless modes must be the particles we see. The electron's mass is $0.5\ MeV$, the proton's is $1\ GeV$, and the W particle has a mass of $80\ GeV$, but the vibrational modes have mass/energy on the order of the Planck mass/energy $10^{19}\ GeV$. Presumably these particles appeared only at the beginning of the universe.
- We need supersymmetry: every particle must have a super partner.

Example 1. Type IIA. Low energy (massless strings). In ten dimensions specify a metric $g_{\mu\nu}$, a scalar φ, an anti-symmetric tensor $B_{\mu\nu}$, a vector potential A_μ, and an anti-symmetric tensor $A_{\mu\nu\rho}$. As usual $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$; we can do a similar thing for $B_{\mu\nu}$ and $A_{\mu\nu\rho}$ to get anti-symmetric tensors

$$\begin{aligned} H_{\mu\nu\rho} &= \partial_\mu B_{\nu\rho} + \partial_\rho B_{\mu\nu} + \partial_\nu B_{\rho\mu} \\ F_{\mu\nu\rho\sigma} &= \partial_\mu A_{\nu\rho\sigma}. \end{aligned}$$

Also we can anti-symmetrize $F_{\mu\nu\rho\sigma} + H_{\mu\nu\rho}A_\sigma$ to get a $G_{\mu\nu\rho\sigma}$. The action is

$$\begin{aligned} S_{10} &= \frac{1}{16\pi G_{10}} \int d^{10}x\sqrt{-g} \left(\begin{array}{c} R - \frac{1}{2}\left(\partial_\mu\varphi\right)^2 - \frac{1}{12}e^{\varphi/2}H_{\mu\nu\rho}H^{\mu\nu\rho} - \\ \frac{1}{4}e^{5\varphi/2}\left(F_{\mu\nu}F^{\mu\nu} - \frac{1}{12}G_{\mu\nu\rho\sigma}G^{\mu\nu\rho\sigma}\right) \end{array} \right) \\ &\quad - \frac{1}{32\pi G_{10}} \int \epsilon^{\mu_1 \cdots \mu_{10}} B_{\mu_1\mu_2} F_{\mu_3\cdots\mu_6} F_{\mu_7\cdots\mu_{10}} \end{aligned}$$

where ϵ is the Levi-Civita tensor (zero if any indices are repeated, ± 1 if the permutation of the indices is even (odd)).
In eleven dimensions the situation is simpler. We only require an anti-symmetric tensor A_{ABC} with anti-symmetric F_{ABCD} as above. The action is

$$S_{11} = \frac{1}{16\pi G_{11}} \int d^{11}x\sqrt{-g^{(11)}} \left(R^{(11)} + \frac{1}{18}F_{ABCD}F^{ABCD} \right).$$

If the tenth dimension is tightly wrapped, we can partition the matrix $g^{(11)}_{AB}$ into a 9 by 9 matrix $g^{(10)}_{\mu\nu}$, a 9 by 1 column A_μ (and the same 1 by 9 row) and a scalar φ:

$$g^{(11)}_{AB} = \begin{pmatrix} g^{(10)}_{\mu\nu} & A_\mu \\ A_\mu & \varphi \end{pmatrix}.$$

If we restrict the indices of A_{ABC} to be from 0 to 9, we get a 10-dimensional tensor $A_{\mu\nu\rho}$ and setting $B_{\mu\nu} = A_{\mu\nu(10)}$, we obtain all the fields for the 10-dimensional case and $S^{(11)}$ gives $S^{(10)}$.
Example 2. Type IIB. p-brane solutions. We are given an anti-symmetric tensor $A_{\mu_1\cdots\mu_{p+1}}$ and a metric

$$ds^2 = -H^{(p-7)/8}\left(-f(r)dt^2 + d\mathbf{y}^2\right) + H^{(p+1)/8}\left(\frac{1}{f(r)}dr^2 + r^2 d\Omega^2_{8-p}\right)$$

where

$$H = 1 + \frac{\rho^{7-p}}{r^{7-p}},\ e^{2\varphi} = H^{(3-p)/2},\ f(r) = 1 - \frac{r_+^{7-p}}{r^{7-p}},\ \text{and } \mathbf{y} \in \mathbb{R}^p.$$

This would be a black hole except for the presence of $d\mathbf{y}$. It appears as if a black hole is stretched through other dimensions and is called a black brane around $\langle \mathbf{y} \rangle$.
In the extremal limit $r_+ \to 0$,

$$ds^2 = -H^{(p-7)/8}\left(-dt^2 + d\mathbf{y}^2\right) + H^{(p+1)/8}d\mathbf{x}^2$$

where $\mathbf{x} \in \mathbb{R}^{9-p}$. As $r \to 0$, H has a singularity and

$$ds^2 \approx \left(\frac{\rho}{r}\right)^{-(7-p)^2/8}\left(-dt^2 + d\mathbf{y}^2\right) + \left(\frac{\rho}{r}\right)^{(7-p)(p+1)/8} d\mathbf{x}^2.$$

When $p = 3$, we get something special, the dilaton φ is constant and

$$\begin{aligned} ds^2 &\approx \frac{r^2}{\rho^2}\left(-dt^2 + d\mathbf{y}^2\right) + \frac{\rho^2}{r^2}d\mathbf{x}^2 \\ &= \frac{r^2}{\rho^2}\left(-dt^2 + d\mathbf{y}^2\right) + \frac{\rho^2}{r^2}dr^2 + \rho^2 d\Omega^2_5. \end{aligned}$$

The last term is just the five sphere S^5 and r would not fall out if $p \neq 3$. The other terms are the anti-deSitter space AdS_5 (3 spatial and 1 time dimension). The situation is similar to two dimensional conformal theory on the boundary giving quantum mechanics.