A New Look at Pictures in Quantum Information

Arthur Jaffe and Zhengwei Liu

Workshop on Higher Category Approach to Certifiably Correct Quantum Information Processing Systems

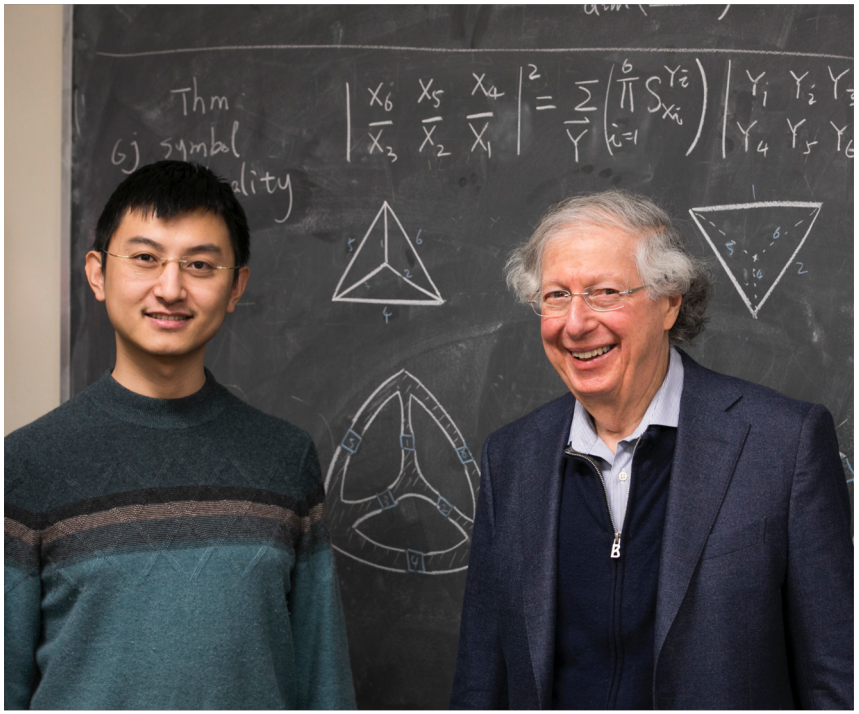
College Park, February 4, 2019

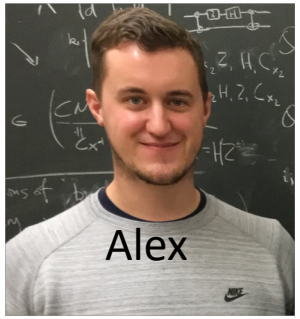
Outline

- The Picture Language Project at Harvard
- Mathematical Background
- Some Problems Solved
- Some Interesting Directions for the Future
- Some Work in Progress (Zhengwei)

The Picture Language Project at Harvard

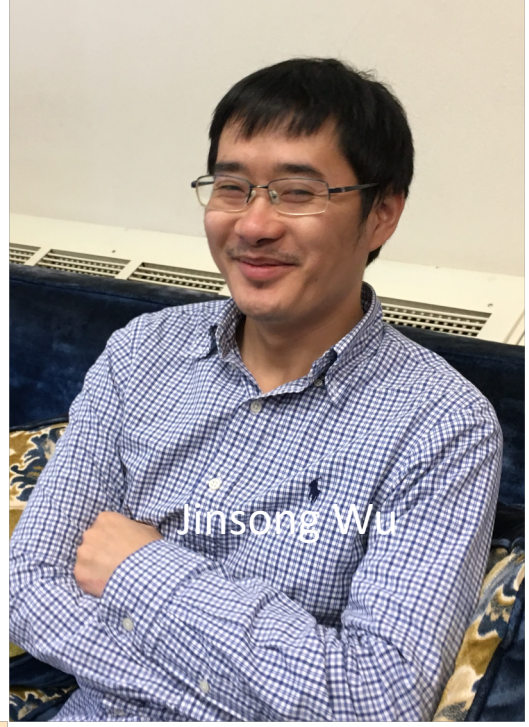
Mathematics, Physics, Quantum Information






Youwei Zhou


LOWELL HOUSE
Kaifeng Bu
Weichen Gu


Jinsong Wu


Xun Gao
Shengtao Wang
Sirui Lu


Yunxiang Ren

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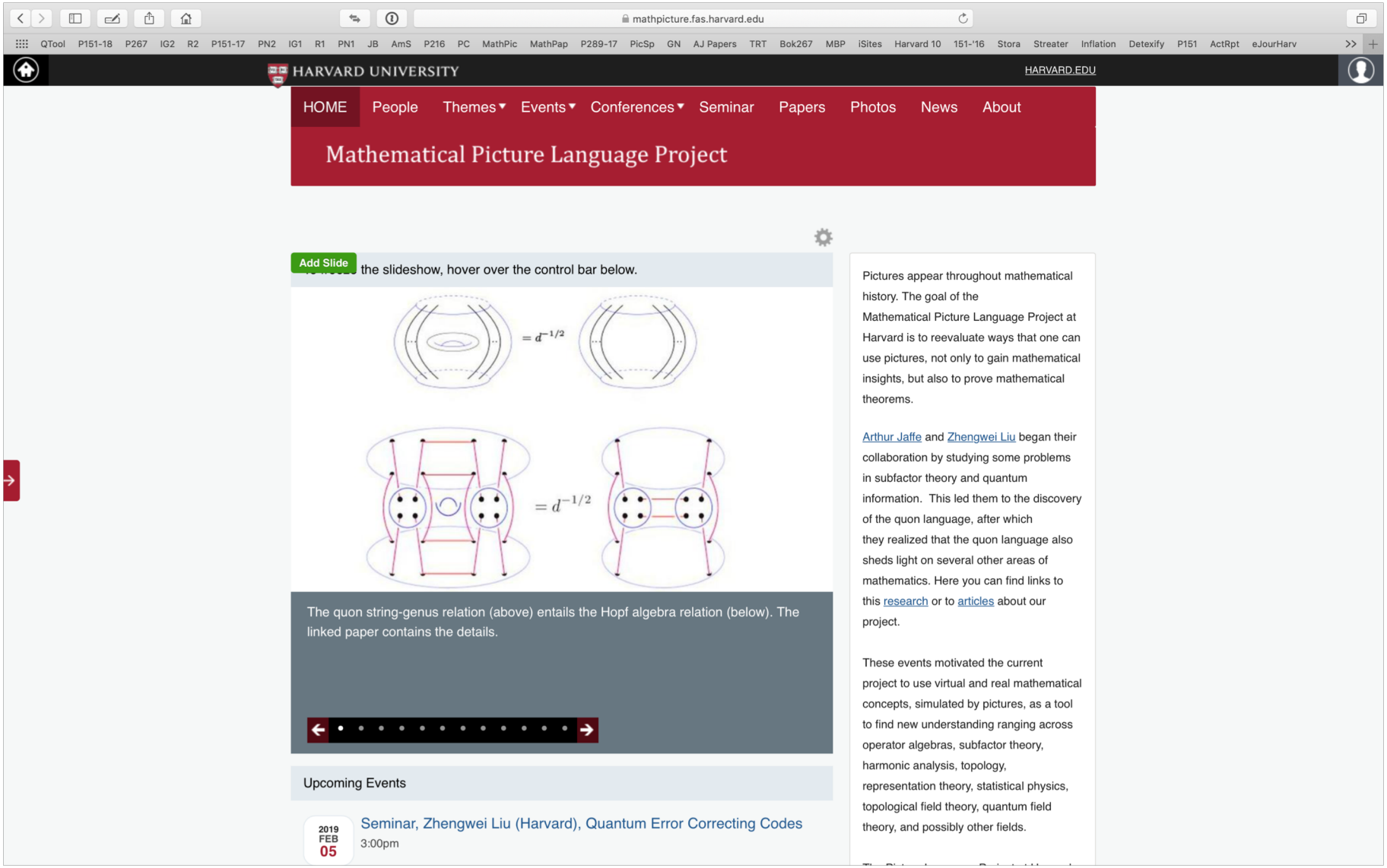

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Mathematical Picture Language Project

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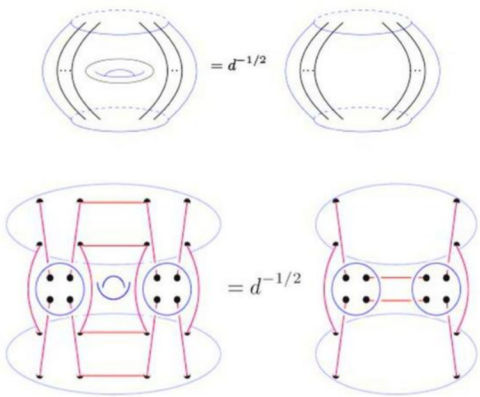

The quon string-genus relation (above) entails the Hopf algebra relation (below). The linked paper contains the details.

Upcoming Events

2019 FEB 05 Seminar, Zhengwei Liu (Harvard), Quantum Error Correcting Codes 3:00pm

Pictures appear throughout mathematical history. The goal of the Mathematical Picture Language Project at Harvard is to reevaluate ways that one can use pictures, not only to gain mathematical insights, but also to prove mathematical theorems.

Arthur Jaffe and Zhengwei Liu began their collaboration by studying some problems in subfactor theory and quantum information. This led them to the discovery of the quon language, after which they realized that the quon language also sheds light on several other areas of mathematics. Here you can find links to this research or to articles about our project.

These events motivated the current project to use virtual and real mathematical concepts, simulated by pictures, as a tool to find new understanding ranging across operator algebras, subfactor theory, harmonic analysis, topology, representation theory, statistical physics, topological field theory, quantum field theory, and possibly other fields.

Lots of Activity: 43 Recent Papers

Safari File Edit View History Bookmarks Window Help 100% Sun Feb 3 6:40:29 Arthur Jaffe

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Links to Recent Papers by Participants that are Related to the Project (Links to posters are at the bottom of this page.)

[43] **Arthur Jaffe and Zhengwei Liu**, Refelction Positivity for Levin-Wen models, arXiv:1901.10662

[42] **Zhengwei Liu**, **William Norledge**, and **Adrian Ocneanu**, The adjoint braid arrangement as a combinatorial Lie algebra via the Steinmann relations, arXiv:1901.03243

[41] Lu Li, **Kaifeng Bu**, and Zi-Wen Liu, Quantifying the resourse content of quantum channels: An operational approach, arXiv:1812.02572

[40] **Kaifeng Bu** and Dax Enshan Koh, Classical simulation of quantum circuits by half Gauss sums, arXiv:1812.00224

[39] **Fei Wei**, Arithmetic functions of natural numbers and its application, arXiv:1811.11000

[38] **Kaifeng Bu**, **Weichen Gu**, and **Arthur Jaffe**, Duality of graph invariants, arXiv:1810.08633

[37] **Fei Wei**, Disjointness of Möbius from asymptotically periodic functions, arXiv:1810.07360

[36] **Yunxiang Ren**, From skein theory to presentations for Thompson group, Journal of Algebra, 498(1) 2018, 178-196

[35] Ryuji Takagi, Bartosz Regula, **Kaifeng Bu**, Zi-Wen Liu, and Gerardo Adesso, Operational advantage of quantum resources in subchannel discrimination, arXiv:1809.01672. To appear in *Physical Review Letters*.

[34] Jordan Nikkel and **Yunxiang Ren**, On Jones Subgroup of R. Thompson's Group T, arXiv:1710.06972, *International Journal of Algebra and Computation*, **28(5)** (2018), 877-903.

[33] **Kaifeng Bu**, **Arthur Jaffe**, **Zhengwei Liu** and **Jinsong Wu**, De Finetti Theorems for Braided Parafermions, arXiv:1805.05990

[32] **Zhengwei Liu** and **Jinsong Wu**, Antisymmetric Characters and Fourier Duality, arxiv:1805.04783

[31] **Zhengwei Liu** and **Jinsong Wu**, Extremal Pairs of Young's Inequality for Kac Algebras, *Pacific Journal of Mathematics*, 295(1) (2018), 103-121.

[30] **Arthur Jaffe**, Reflection Positivity Then and Now, arXiv:1802.07880

[29] **William Norledge**, Singer cyclic lattices of type M, arXiv:1711.09332.

[28] **William Norledge**, Quotients of buildings by groups acting freely on chambers, arXiv: 1711.09330.

[27] **William Norledge**, Quotients of buildings as W-groupoids, arXiv:1710.06968.

[26] Keshab Chandra Bakshi, Sayan Das, **Zhengwei Liu**, and **Yunxiang Ren**, An angle between intermediate subfactors and its rigidity, arXiv:1710.00285, to appear in *Transactions of the AMS*.

[25] **Zhengwei Liu**, Scott Morrison, and David Penneys, Lifting shadings on symmetrically self-dual subfactor planar algebras, arXiv:1709.05023, to appear in *Contemporary Mathematics*.

[24] **Arthur Jaffe** and **Zhengwei Liu**, A Mathematical Picture Language Program, *Proceedings of the National Academy of Sciences* 115(1) (2018), 81–86.

[23] **Zhengwei Liu**, Quon Language: Surface Algebras and Fourier Duality, arXiv:1707.03995.

[22] **Chunlan Jiang**, **Zhengwei Liu,** and **Jinsong Wu**, Block maps and Fourier analysis, arXiv:1706.03551.

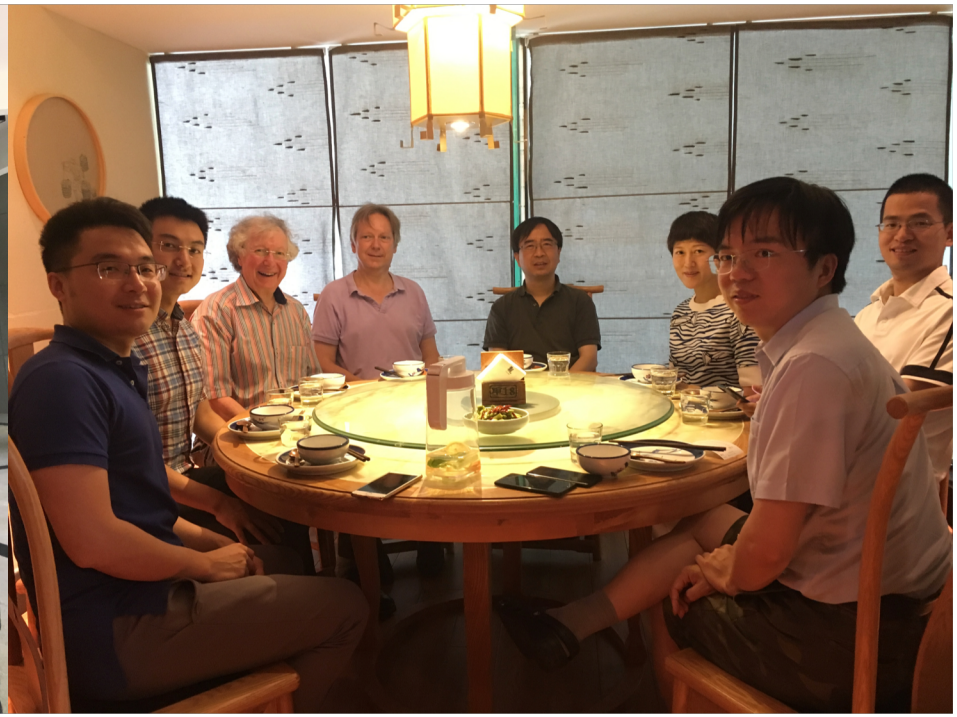
[21] **Zhengwei Liu** and Feng Xu, Jones-Wassermann subfactors for modular tensor categories, arXiv:1612:08573.

[20] Corey Jones, **Zhengwei Liu**, and **Yunxiang Ren**, Classification of Thurston-Relation Subfactor Planar Algebras, arXiv:1606.00779, to appear in *Quantum Topology*.

[19] **Arthur Jaffe**, **Zhengwei Liu**, and **Alex Wozniakowski**, Holographic Software for Quantum Networks, *Science China Mathematics* 61(4) (2018), 593-626.

July 2, 2018, Shanghai

Institute of Jian-Wei Pan



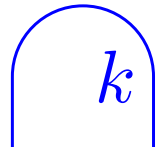
Institute for Quantum Physics and Communication in Pudong

University of Science and Technology (USTC) and the Chinese Academy of Sciences (CAS)

US $12 Billion / 5 years in 1 Institute!!

Mathematical Background
2 Charged-String Language (2D)

Qubit Vectors

$|k\rangle = 2^{-\frac{1}{4}}$ 

k is a "charge" label in Z_d . *k = 0, …, d-1*

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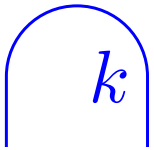
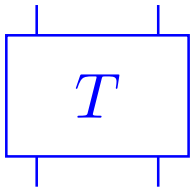
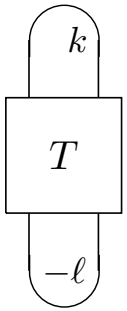
$|k\rangle = 2^{-\frac{1}{4}}$ 

One-Qudit Transformations



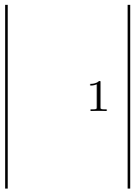


= $\langle \ell | \, T | k \rangle$

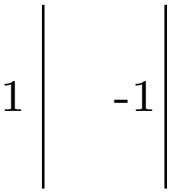
k is a "charge" label in Z_d . *k = 0, …, d-1*

Pauli I, X, Y, Z

$I =$

$X =$ 

$Y =$

$Z =$ 

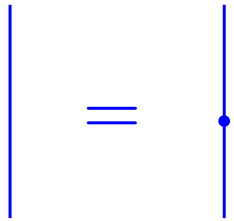
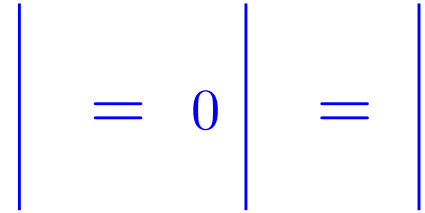
Vectors 

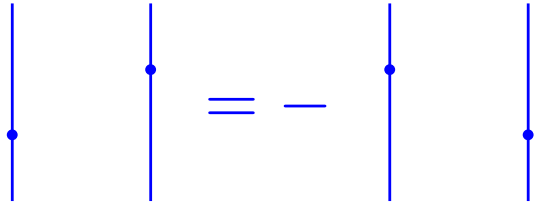
are an eigenbasis for Pauli Z.

Majorana Algebra (Case d=2)

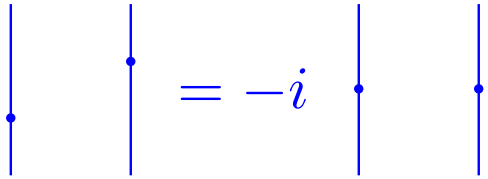
Resolution of single strands (for Qubits)

$$c = c^* = c^{-1}$$

$c =$  $I = c^2 =$ 

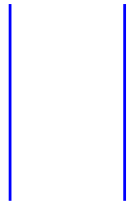
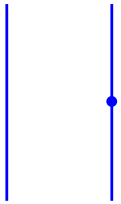
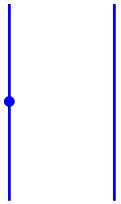
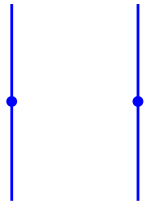
Parasisotopy:  $YX = -XY$

Twisted Tensor Product: $(c \otimes I)(I \otimes c) = -(I \otimes c)(c \otimes I)$

Interpolation: 

$$YX = -iZ$$
$$XYZ = i$$

Qubit Pauli Matrices

$I =$  $X =$  $Y =$  $Z =$ 

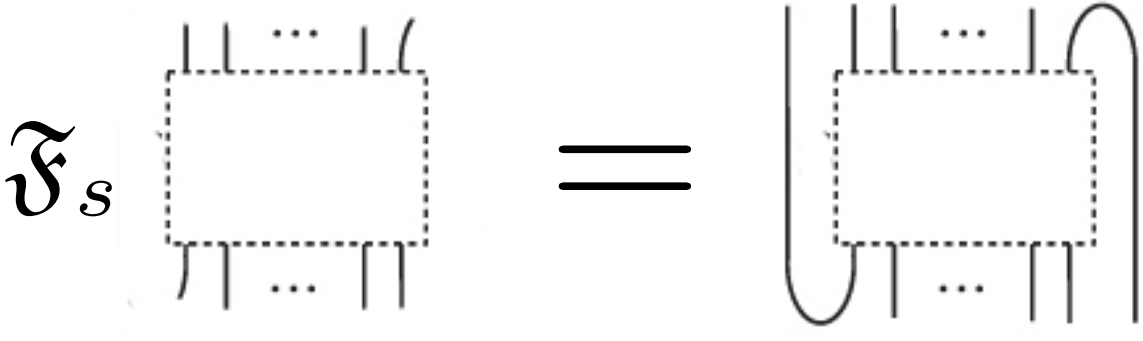
Fourier Transform (FT) on Z_d

$$(Ff)(k) = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} q^{k\ell} f(\ell)$$

Fourier transform on projections (neutral matrix units) is:

$$F \begin{array}{c} \cup_{-k} \\ \cap_{k} \end{array} = \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} q^{k\ell} \begin{array}{c} \cup_{-\ell} \\ \cap_{\ell} \end{array}$$

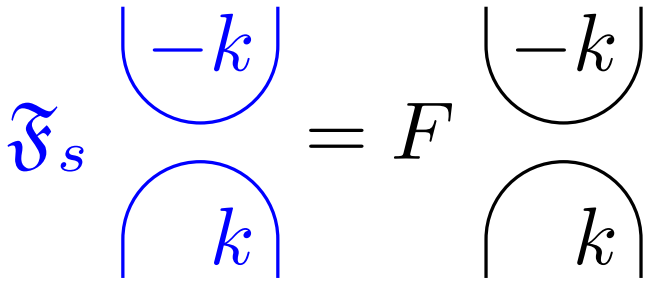
String Fourier Transform (SFT)



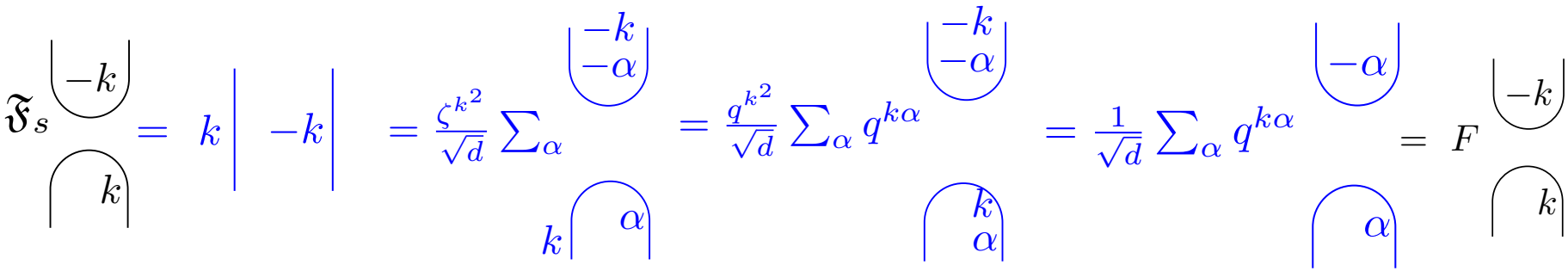
90° rotation of inputs and outputs on a 1-qubit transformation

Theorem. (SFT generalizes FT)

On neutral, one-qudit transformations,



One line proof of the theorem:



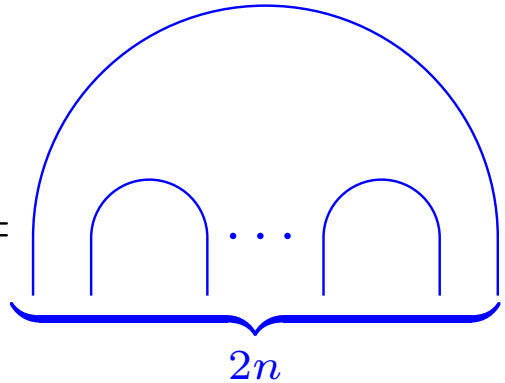
Move –k up.
Insert resolution of identity.

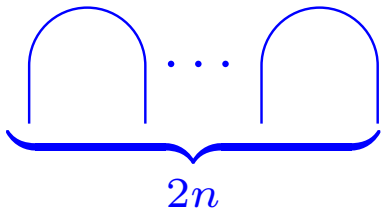
Move k across cap. α ➔α-k

Some Results

- Translation between Algebraic and Diagrammatic Protocols
- Entanglement by SFT
- Topological Design of Protocols

Generalized Bell State: Max
Maximally Entangled n-qubit state

$2^{\frac{n}{4}} |\mathrm{Max}\rangle_n =$ 

$= \mathfrak{F}_s$ 

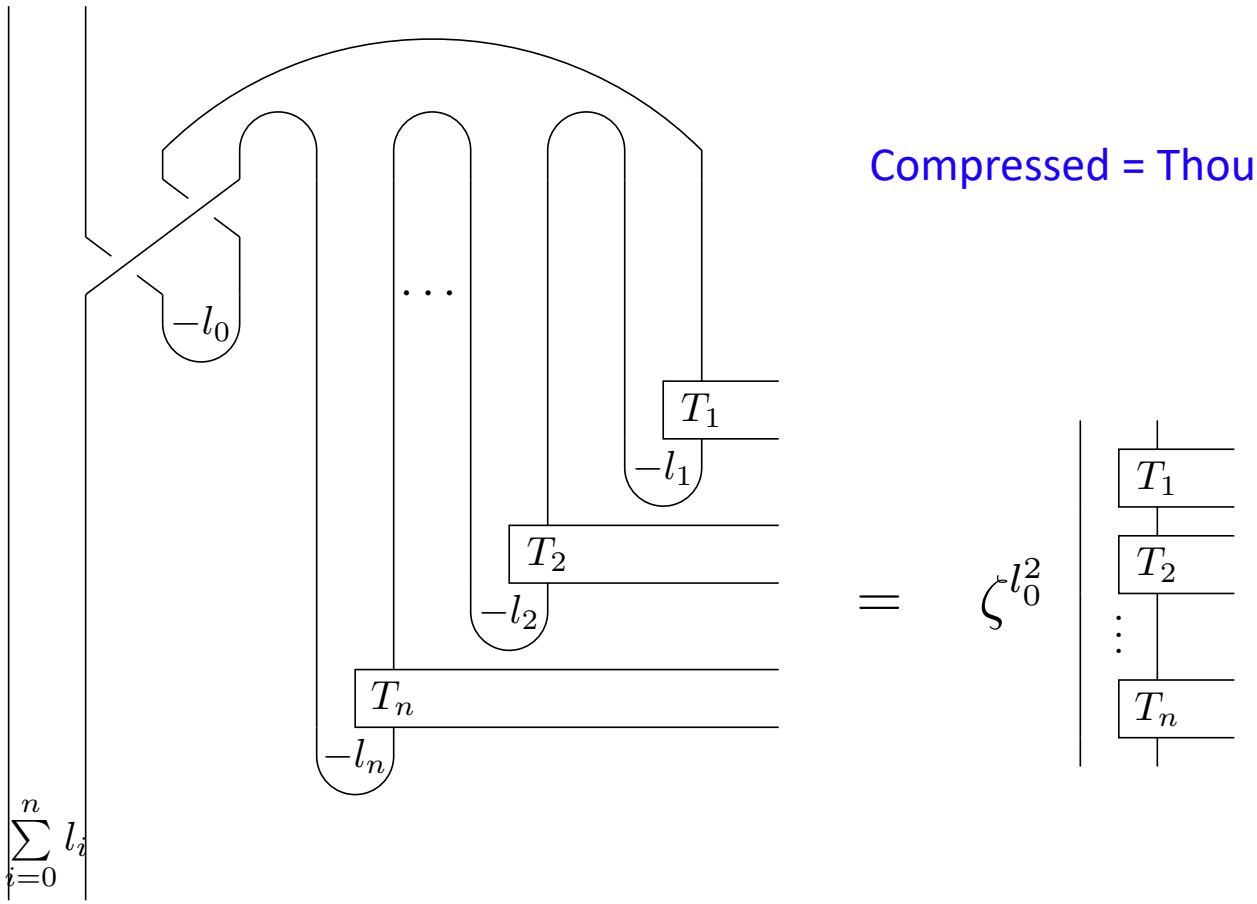
Kauffman entanglement by braid; SFT maximal entanglement.

$$|\mathrm{Max}\rangle_2 = |\mathrm{GHZ}\rangle_2 = \frac{1}{2^{\frac{1}{2}}} \left(|0,0\rangle + |1,1\rangle\right)$$

$$|\mathrm{Max}\rangle_n = F^{\otimes n} |\mathrm{GHZ}\rangle_n$$

Topological Design

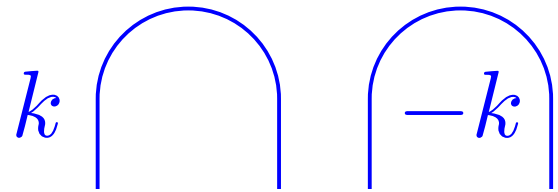
Multipartite Compressed Teleportation



Compressed = Though Line

Quon (4 strings, 3D)

$$2^{\frac{1}{2}}|k\rangle =$$



one cap replaced by neutral pair
2-dimensional neutral subspace in 4-dimensional space

Quon
Pauli I, X, Y, Z

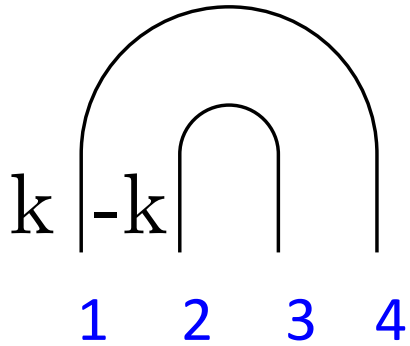
$$I = \begin{vmatrix} & & & \end{vmatrix} \qquad X = \begin{vmatrix} 1 & & & \\ & & & \\ & & & -1 \end{vmatrix} \qquad Y = \begin{vmatrix} -1 & & & \\ & & 1 & \\ & & & \end{vmatrix} \qquad Z = \begin{vmatrix} 1 & & & \\ & -1 & & \\ & & & \end{vmatrix}$$

$$XYZ = \zeta\gamma\,, \qquad \gamma = \begin{vmatrix} 1 & & & \\ & -1 & & \\ & & 1 & \\ & & & -1 \end{vmatrix}$$

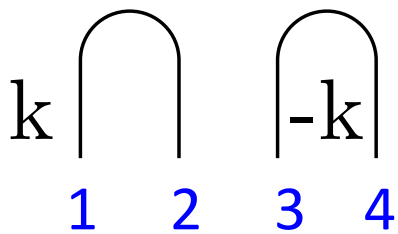
Must restrict to $\gamma = 1$ eigenspace.

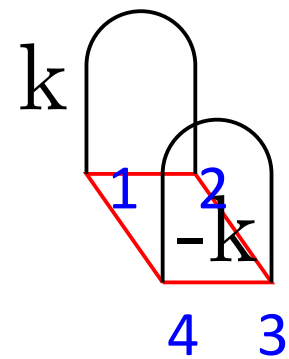
Quon X, Y, and Z Bases

X Eigen Basis

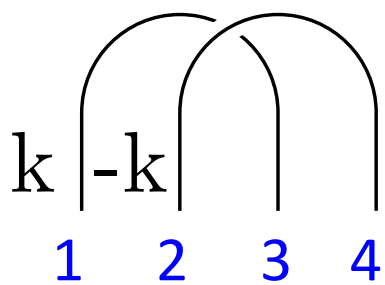


Z Eigen Basis

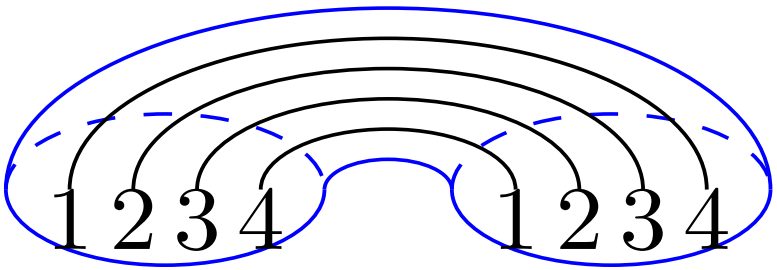




Y Eigen Basis



GHZ_2 and Max_2 in Quon



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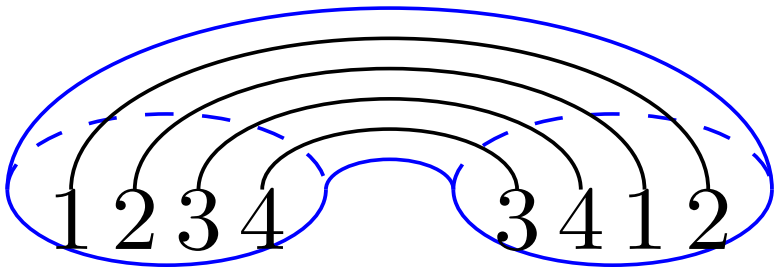

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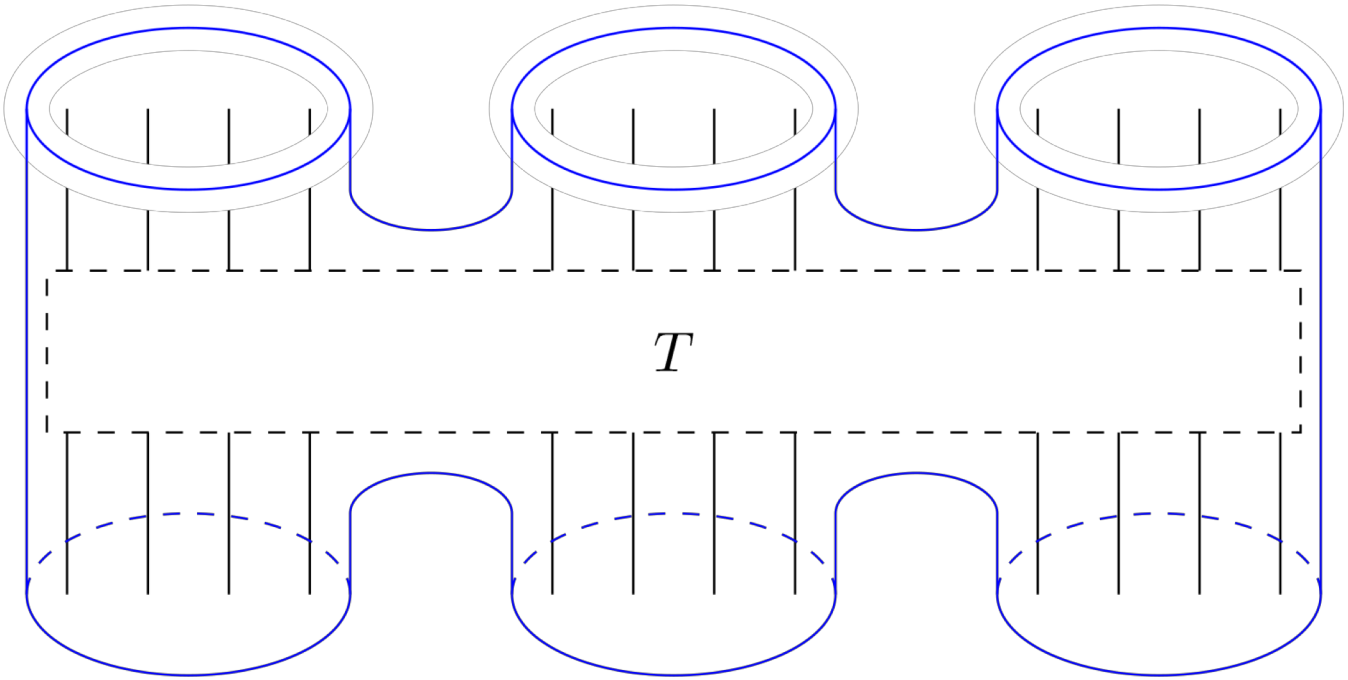
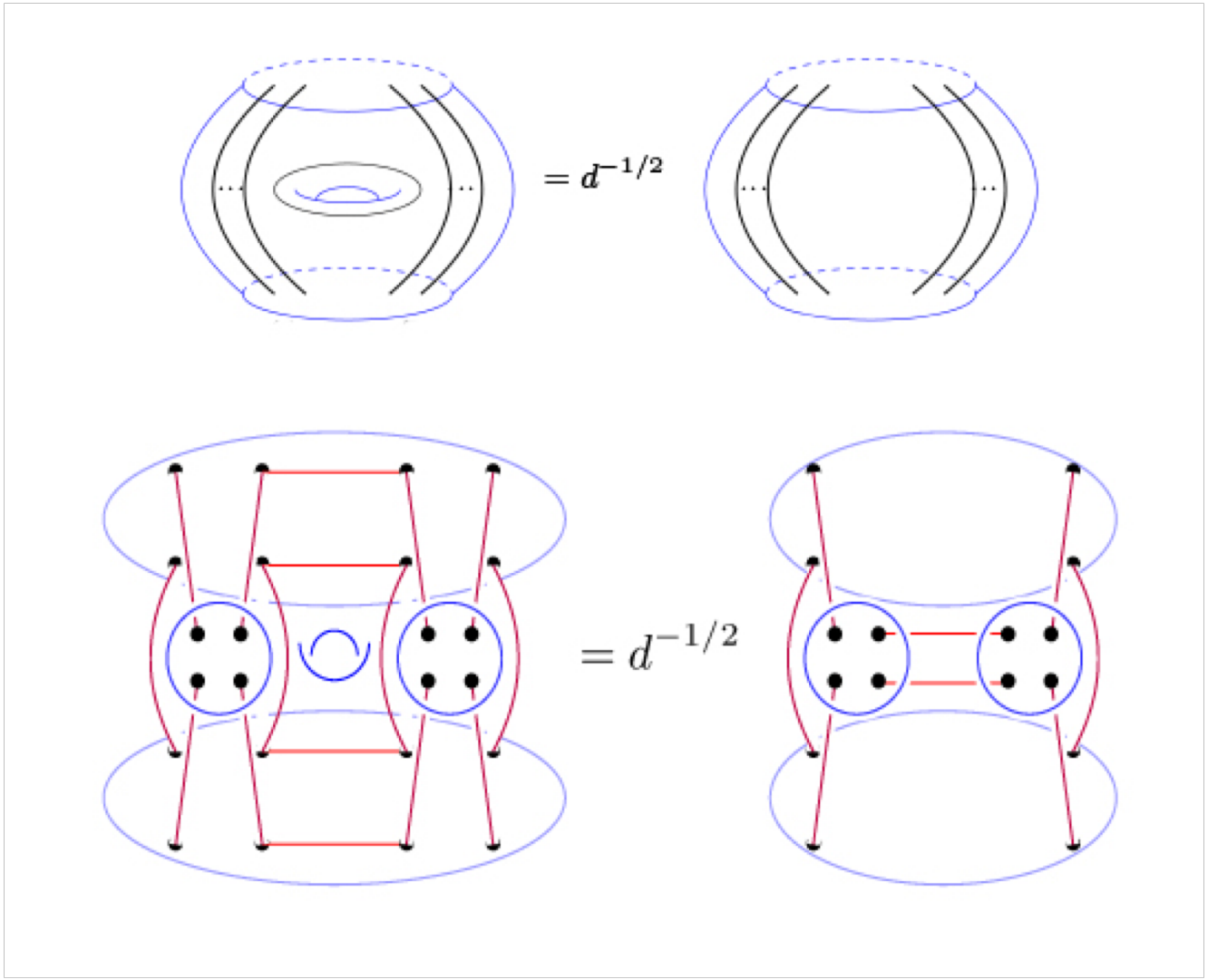
3-Quon to 3-Quon Transformation



TQFT + Strings

String-Genus Relation

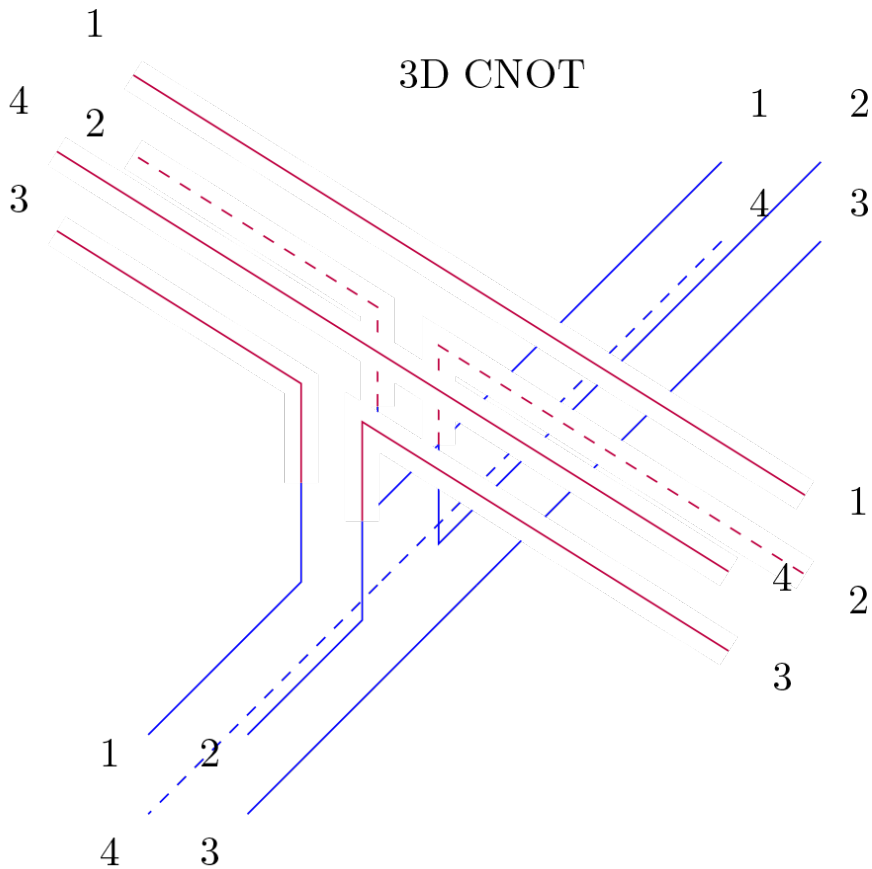


Some Results

Zhengwei Liu involved in all

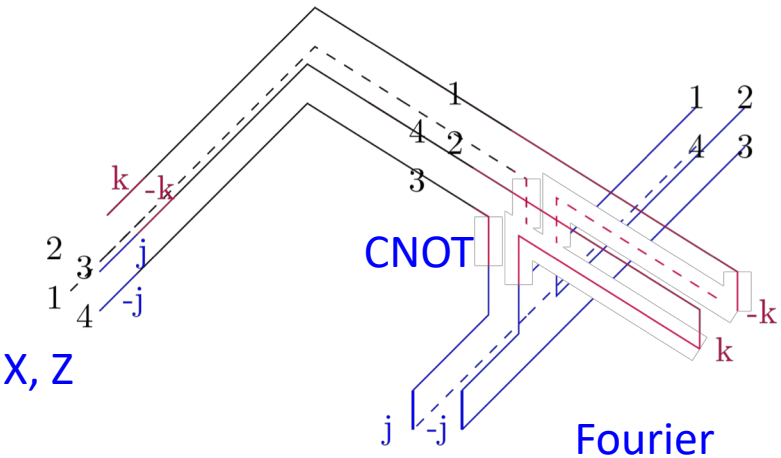
- Topological Interpretation of Quantum CNOT
- Beautiful Teleportation Diagram
- Inequalities for Quantum Fourier Analysis
- Algebraic Identities for 6j Symbols of Quantum groups
- New proof of Verlinde fusion relations
- DeFinetti Theorem for Double Braiding
- SFT = S-Matrix

Topological CNOT

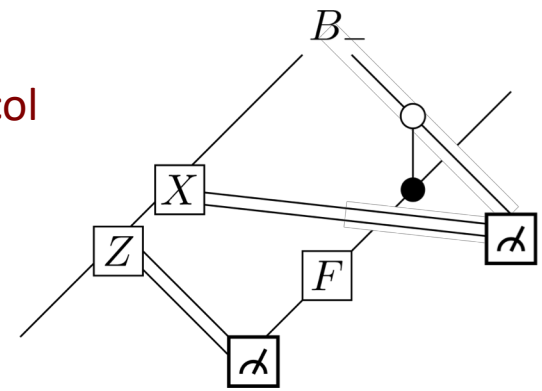


Teleportation Protocol in 3D Quon Language

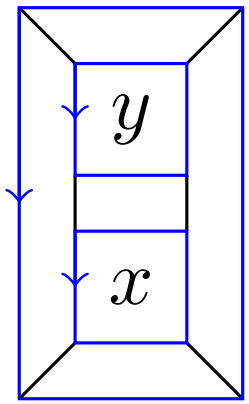
3D Quon Language



Bennett et al. algebraic protocol



Two Multiplications



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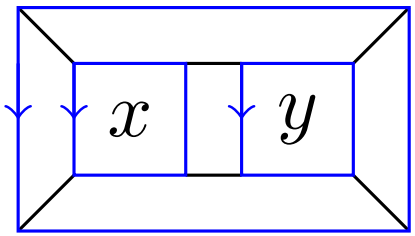

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xy multiplication $\qquad$ $x \star y$ convolution

$$\mathfrak{F}_S(xy) = \mathfrak{F}_S(x) * \mathfrak{F}_S(y)$$

Inequalities: Hausdorf-Young, Young, …..

Uncertainty Principles: Generalize classical

Heisenberg, Hardy, Beckner, Tao, ,….

6j Identites

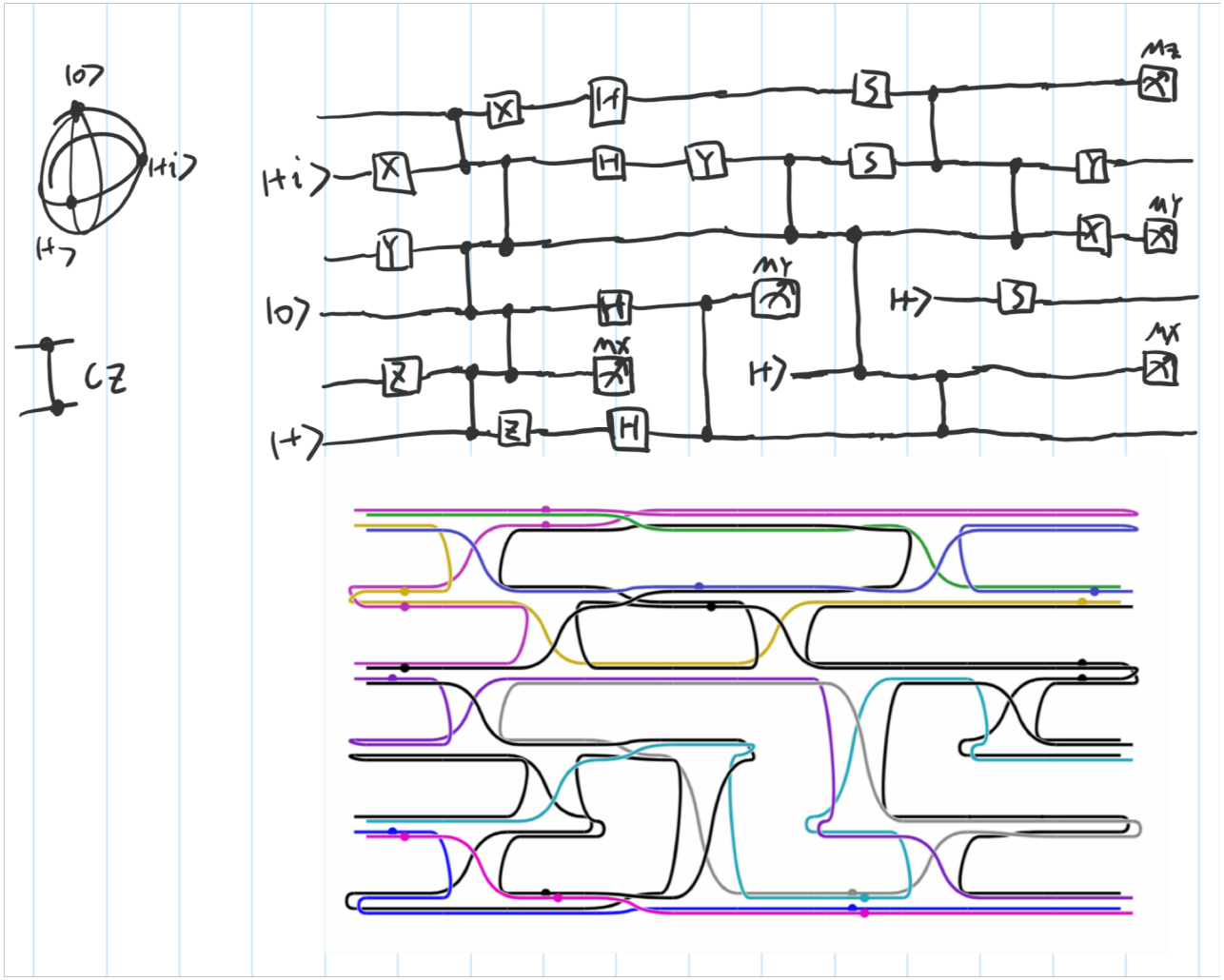
- 6j symbol corresponds to a tetrahedron
- Duality of tetrahedron is a tetrahedron
- 6j identities for quantum groups from Max
  ←→ GHZ (Liu). $\overline{X}$ dual to X in an MTC.

$$\left|\left(\frac{X_6}{X_3}\,\frac{X_5}{X_2}\,\frac{X_4}{X_1}\right)\right|^2 = \sum_{\vec{Y}}\left(\prod_{k=1}^{6} S_{X_k}^{Y_k}\right)\left|\begin{pmatrix} Y_1 Y_2 Y_3 \\ Y_4 Y_5 Y_6 \end{pmatrix}\right|^2 .$$

Open Questions

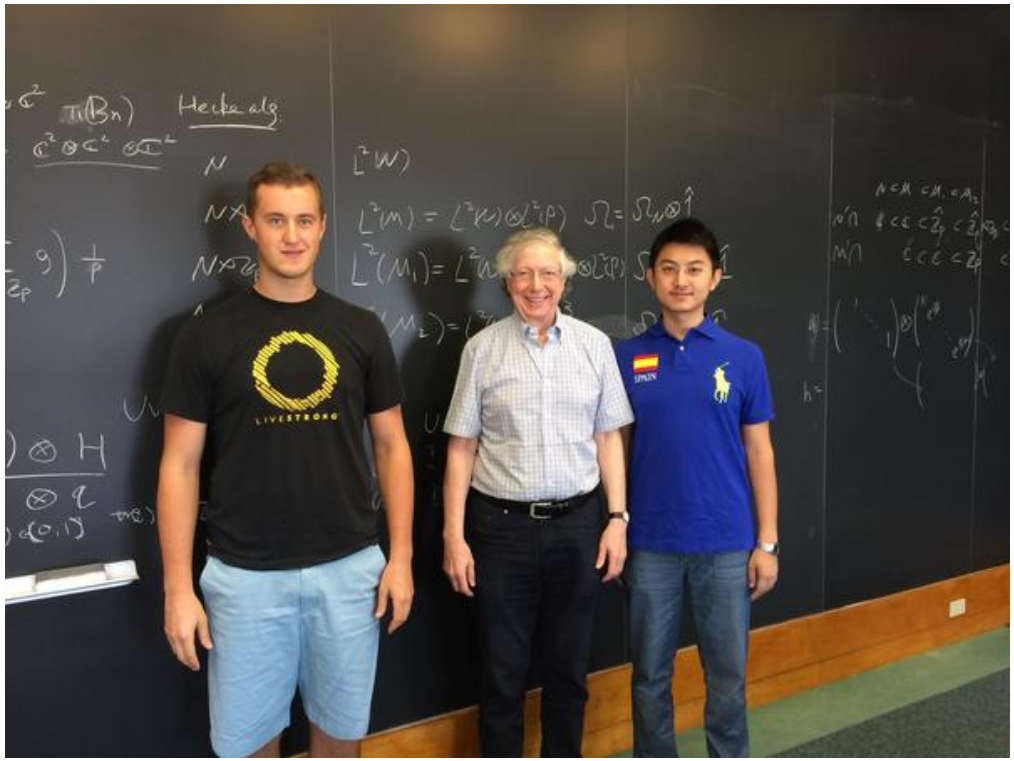
- Qtool for visualization of quon model
- Approach to quantum error correction
- New Algorithms
- Exactly soluble Statistical Mechanics Models

Youwei Zhou's QTool



Quon Language

In the joint paper with Arthur Jaffe and Alex Wozniakowski (PNAS 2017), we introduce the **quon language** as a 3D picture language for quantum information. In this talk, we restrict the quon language on the 2D surfaces, which is a surface algebra.



The quon Language refines the tensor network approach to quantum information. A single string in tensor network is refined as four strings in a cylinder in the quon language. A Bosonic particle can be decomposed into a particle-antiparticle pair. In this way, we can see further internal symmetry and duality pictorially.

A new point of view can give new insight!


2019
新年快樂
Happy New Year